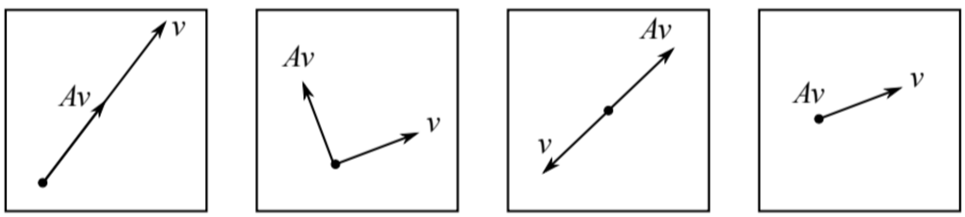
Name: ______________________________ Recitation Section: ____________

Math 1553 Quiz 7: 5.1, 5.2 (10 points, 10 minutes)

Solutions

Show your work and justify answers where appropriate. If you write the correct answer without sufficient work or justification, you will receive little or no credit.

1. Under each picture, write the *eigenvalue* being depicted (an estimate is fine). If the picture does not show an eigenvector, write NO. You do not need to justify your answer. (Only real numbers are allowed for eigenvalues in this problem)



(a) $\lambda = \frac{1}{2}$ (b) NO (c) $\lambda = -1$ (d) $\lambda = 0$

2. Find the eigenvalues and a basis for each eigenspace of $A = \begin{pmatrix} 0 & -1 \\ 2 & 3 \end{pmatrix}$.

Solution.

The characteristic equation is

$$0 = (-\lambda)(3-\lambda)+2 = \lambda^2 - 3\lambda + 2 = (\lambda-1)(\lambda-2), \qquad \lambda = 1, \quad \lambda = 2$$

$\underline{\text{For } \lambda_1 = 1}$: $A - I = \begin{pmatrix} -1 & -1 \\ 2 & 2 \end{pmatrix} \xrightarrow{\text{rref}} \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$.

Therefore, $(A-2I)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$ if and only if $x_1 = -x_2$, $x_2 = x_2$.

A basis for the 1-eigenspace is $\left\{ \begin{pmatrix} -1 \\ 1 \end{pmatrix} \right\}$.

$\underline{\text{For } \lambda_2 = 2}$: $A - 2I = \begin{pmatrix} -2 & -1 \\ 2 & 1 \end{pmatrix} \xrightarrow{\text{rref}} \begin{pmatrix} 1 & 1/2 \\ 0 & 0 \end{pmatrix}$.

Therefore, $(A-2I)\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$ if and only if $x_1 = -\frac{x_2}{2}$, $x_2 = x_2$.

A basis for the 2-eigenspace is $\left\{ \begin{pmatrix} -1/2 \\ 1 \end{pmatrix} \right\}$ (or $\left\{ \begin{pmatrix} -1 \\ 2 \end{pmatrix} \right\}$, etc.)

3. True or false, 1 point each. If the statement is *always* true, answer true. Otherwise, answer false.

a) If A and B are 2×2 matrices and each has characteristic polynomial $(\lambda - 1)^2$, then A is similar to B. TRUE FALSE

b) If A is an 3×3 matrix and 2 is an eigenvalue of A, then 10 is an eigenvalue of $5A$. TRUE FALSE

Solution.

a) False. For example, $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ both have characteristic polynomial $(\lambda - 1)^2$, but are not similar. In fact, the only matrix similar to the identity matrix is the identity matrix itself.

b) True. If $v \neq 0$ and $Av = 2v$, then $(5A)v = 5Av = 5(2v) = 10v$, so 10 is an eigenvalue of $5A$.