Name:____________________ Recitation Section:__________

Math 1553 Quiz 6: chapter 3 (10 points, 10 minutes)

Solutions

1. Suppose A is a 3×3 matrix with $\det(A) = -1$, and let $T : \mathbf{R}^3 \to \mathbf{R}^3$ be the transformation $T(x) = Ax$. Which of the following statements *must* be true?
Circle *all* that apply. You do not need to justify your answers.

(a) A is the negative of the identity matrix

(b) $\det(-A) = 1$.

(c) For every 3×3 matrix B, we have $\det(AB) = -\det(B)$.

(d) If S is a subset of $\mathbf{R}^3$ with volume 10, then the volume of $T(S)$ is 10.

Solution.

(b), (c), and (d). Note $\det\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = -1$, so A is not necessarily $-I_3$.

If $\det(A) = -1$ then $\det(-A) = (-1)^3 \det(A) = (-1)(-1) = 1$, so (b) is true.
If B is any 3×3 matrix, then $\det(AB) = \det(A)\det(B) = -\det(B)$, so (c) is true.
If $\mathrm{Vol}(S) = 10$ then $\mathrm{Vol}(T(S)) = |\det(A)| \cdot \mathrm{Vol}(S) = 1 \cdot 10 = 10$, so (d) is true.

2. (3 points each)

a) Find $\det\begin{pmatrix} 0 & 0 & 3 & -1 \\ 4 & 2 & -1 & 1 \\ 3 & 0 & 1 & 2 \\ 0 & 0 & 1 & 4 \end{pmatrix}$.

b) Suppose $\det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = 2$. Find $\det\begin{pmatrix} d & e & f \\ 3a+d & 3b+e & 3c+f \\ g-4a & h-4b & i-4c \end{pmatrix}$.

Solution.

a) $\det\begin{pmatrix} 0 & 0 & 3 & -1 \\ 4 & 2 & -1 & 1 \\ 3 & 0 & 1 & 2 \\ 0 & 0 & 1 & 4 \end{pmatrix} = 2(-1)^4 \det\begin{pmatrix} 0 & 3 & -1 \\ 3 & 1 & 2 \\ 0 & 1 & 4 \end{pmatrix} = 2\cdot 3(-1)^3 \det\begin{pmatrix} 3 & -1 \\ 1 & 4 \end{pmatrix} =$
$-6(12+1) = -78$. We used the cofactor expansion along column 2, then along column 1.

b) We multiply row 1 by 3 and swap the first two rows, multiplying the determinant by -3. The row-replacements do nothing to change the determinant, so the determinant of the final matrix is $2(-3) = -6$.