Math 1553 Worksheet §2.8 (and some 2.9)

1. Find bases for the column space and the null space of

$$A = \begin{pmatrix} 0 & 1 & -3 & 1 & 0 \\ 1 & -1 & 8 & -7 & 1 \\ -1 & -2 & 1 & 4 & -1 \end{pmatrix}.$$

2. Consider the following vectors in $\mathbf{R}^3$:

$$b_1 = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} \qquad b_2 = \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} \qquad u = \begin{pmatrix} 1 \\ 10 \\ 7 \end{pmatrix}$$

Let $V = \operatorname{Span}\{b_1, b_2\}$.

a) Explain why $\mathcal{B} = \{b_1, b_2\}$ is a basis for V.

b) Determine if u is in V.

c) Find a vector b_3 such that $\{b_1, b_2, b_3\}$ is a basis of $\mathbf{R}^3$.

3. For (a) and (b), answer "yes" if the statement is always true, "no" if it is always false, and "maybe" otherwise.

a) If A is an $n \times n$ matrix and Col $A = \mathbf{R}^n$, then $Ax = 0$ has a nontrivial solution.

b) If A is an $m \times n$ matrix and $Ax = 0$ has only the trivial solution, then the columns of A form a basis for $\mathbf{R}^m$.

c) Give an example of 2×2 matrix whose column space is the same as its null space.

4. In each case, determine whether the given set is a subspace of $\mathbf{R}^4$. If it is a subspace, justify why. If it is not a subspace, state a subspace property that it fails.

a) $V = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \text{ in } \mathbf{R}^4 \;\middle|\; x + y = 0 \text{ and } z + w = 0 \right\}$

b) $W = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \text{ in } \mathbf{R}^4 \;\middle|\; xy - zw = 0 \right\}$

5. This problem covers section 2.9. Parts (a), (b), and (c) are unrelated to each other.

a) True or false: If A is a 3×100 matrix of rank 2, then $\dim(\operatorname{Nul} A) = 97$.

b) For u and $\mathcal{B}$ from problem 2, find $[u]_{\mathcal{B}}$ (the $\mathcal{B}$-coordinates of u).

c) Let $\mathcal{D} = \left\{ \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \end{pmatrix} \right\}$, and suppose $[x]_{\mathcal{D}} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$. Find x.