Math 1553 Worksheet §2.1, 2.2, 2.3

1. If A is a 3×5 matrix and B is a 3×2 matrix, which of the following are defined?

a) $A - B$

b) AB

c) $A^T B$

d) $B^T A$

e) A^2

2. Find all matrices B that satisfy

$$\begin{pmatrix} 1 & -3 \\ -3 & 5 \end{pmatrix} B = \begin{pmatrix} -3 & -11 \\ 1 & 17 \end{pmatrix}.$$

3. **a)** If the columns of an $n \times n$ matrix Z are linearly independent, is Z necessarily invertible? Justify your answer.

b) Solve $AB = BC$ for A, assuming A, B, C are $n \times n$ matrices and B is invertible. Be careful!

4. True or false (justify your answer). Answer true if the statement is *always* true. Otherwise, answer false.

a) If A is an $m \times n$ matrix and B is an $n \times p$ matrix, then each column of AB is a linear combination of the columns of A.

b) If A and B are $n \times n$ and both are invertible, then the inverse of AB is $A^{-1}B^{-1}$.

c) If A^T is not invertible, then A is not invertible.

d) If A is an $n \times n$ matrix and the equation $Ax = b$ has at least one solution for each b in $\mathbf{R}^n$, then the solution is *unique* for each b in $\mathbf{R}^n$.

e) If A and B are invertible $n \times n$ matrices, then $A + B$ is invertible and $(A+B)^{-1} = A^{-1} + B^{-1}$.

f) If A and B are $n \times n$ matrices and $ABx = 0$ has a unique solution, then $Ax = 0$ has a unique solution.

5. Suppose A is an invertible 3×3 matrix and

$$A^{-1}e_1 = \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix}, \quad A^{-1}e_2 = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}, \quad A^{-1}e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

Find A.