Math 1553 Worksheet §1.4

For problems 1, 2, and 3 below: The professor in your widgets and gizmos class is trying to decide between three different grading schemes for computing your final course grade. The schemes are based on homework (HW), quiz grades (Q), midterms (M), and a final exam (F). The three schemes can be described by the following matrix A:

$$\begin{array}{c} \\ \text{Scheme 1} \\ \text{Scheme 2} \\ \text{Scheme 3} \end{array} \begin{array}{c} \begin{array}{cccc} \text{HW} & \text{Q} & \text{M} & \text{F} \end{array} \\ \begin{pmatrix} 0.1 & 0.1 & 0.5 & 0.3 \\ 0.1 & 0.1 & 0.4 & 0.4 \\ 0.1 & 0.1 & 0.6 & 0.2 \end{pmatrix} \end{array}$$

1. Suppose that you have a score of x_1 on homework, x_2 on quizzes, x_3 on midterms, and x_4 on the final, with potential final course grades of b_1, b_2, b_3. Write a matrix equation $Ax = b$ to relate your final grades to your scores.

2. Suppose that you end up with averages of 90% on the homework, 90% on quizzes, 85% on midterms, and a 95% score on the final exam. Use Problem 1 to determine which grading scheme leaves you with the highest overall course grade.

3. **a)** Keeping $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ as a general vector, write the augmented matrix $(A \mid b)$.

b) Row reduce this matrix until you reach row echelon form.

c) Looking at the final matrix in (b), what equation in terms of b_1, b_2, b_3 must be satisfied in order for $Ax = b$ to have a solution?

d) The answer to (c) also defines the span of the columns of A. Describe the span geometrically.

e) Solve the equation in (c) for b_1. Looking at this equation, is it possible for b_1 to be the largest of b_1, b_2, b_3? In other words, is it ever possible for the grade under Scheme 1 to be the highest of the three final course grades? Why or why not? Which scheme would you argue for?

4. True or false. If the statement is ever false, answer false. Justify your answer.

a) A matrix equation $Ax = b$ is consistent if A has a pivot in every column.

b) If $Ax = b$ is inconsistent, then b is not in the span of the columns of A.

c) If A is a 5×4 matrix, then the equation $Ax = b$ must be inconsistent for some b in $\mathbf{R}^5$.

5. Find the solution sets of $x_1 - 3x_2 + 5x_3 = 0$ and $x_1 - 3x_2 + 5x_3 = 3$. How do they compare geometrically? You may want to sketch the two planes to see the picture.