Math 1553 Worksheet §1.3, interactive supplement

If you don't have a computer, find someone who does.

1. Let $v_1 = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$ $v_2 = \begin{pmatrix} -2 \\ -3 \\ -1 \end{pmatrix}$ $w = \begin{pmatrix} 2 \\ -4 \\ 8 \end{pmatrix}$.

Question: Is w a linear combination of v_1 and v_2? In other words, is w in $\mathrm{Span}\{v_1, v_2\}$?

a) Formulate this question as a vector equation.

b) Formulate this question as a system of linear equations.

c) Formulate this question as an augmented matrix.

d) Answer the question using the interactive demo.

e) Answer the question using row reduction.

2. Consider the system of linear equations

$$\begin{aligned} x + 2y &= 7 \\ 2x + y &= -2 \\ -x - y &= 4 \end{aligned}$$

Question: Does this system have a solution? If so, what is the solution set?

a) Formulate this question as an augmented matrix.

b) Formulate this question as a vector equation.

c) What does this mean in terms of spans?

d) Answer the question using the interactive demo.

e) Answer the question using row reduction.

3. Consider the vector equation

$$x \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + y \begin{pmatrix} -2 \\ -1 \\ -1 \end{pmatrix} + z \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix} = \begin{pmatrix} -5 \\ -1 \\ -2 \end{pmatrix}.$$

Question: Is there a solution? If so, what is the solution set?

a) Formulate this question as an augmented matrix.

b) Formulate this question as a system of linear equations.

c) What does this mean in terms of spans?

d) Answer the question using the interactive demo.

e) Answer the question using row reduction.

4. Consider the augmented matrix
$$\left(\begin{array}{ccc|c} 2 & -2 & 2 & 0 \\ 1 & -3 & -4 & -9 \\ 3 & -1 & 8 & 9 \end{array}\right)$$

Question: Does the corresponding linear system have a solution? If so, what is the solution set?

a) Formulate this question as a vector equation.

b) Formulate this question as a system of linear equations.

c) What does this mean in terms of spans?

d) Answer the question using the interactive demo.

e) Answer the question using row reduction.

f) Find a **different** solution in parts (e) and (d).

5. Decide if each of the following statements is true or false. If it is true, prove it; if it is false, provide a counterexample.

a) Every set of four or more vectors in $\mathbf{R}^3$ will span $\mathbf{R}^3$.

b) The span of any set contains the zero vector.