Quiz 7, Discrete Math (15 points), Fall 2016

The quiz is 20 minutes. Show your work and justify your answers where appropriate. If you write the correct answer without sufficient work or justification, you will receive little or no credit.

1. (1 point each) Clearly circle your answer (no justification needed here, and no partial credit given).

   (a) Suppose we use Horner's Algorithm to evaluate $f(x) = 2x^2 - x + 4$ when $x = 2$.
   Originally, we set $S = 2$ (we will keep replacing S until it equals $f(2)$).
   True or false: the next step in Horner's Algorithm is to replace $S = 2$ with $S = 4$. TRUE FALSE

   (b) Let A be the set of all functions from $\mathbb{N}$ to $\mathbb{R}$. Define a binary relation $\mathcal{R}$ on A by:

   $$(f, g) \in \mathcal{R} \iff f = \mathcal{O}(g).$$

   Then $\mathcal{R}$ is transitive. TRUE FALSE

   (c) If $f, g : \mathbb{N} \to \mathbb{R}$ are given by $f(n) = n^a$ and $g(n) = n^b$ for some real numbers a and b, then $f \asymp g$ if and only if $a = b$. TRUE FALSE

   (d) If $f, g : \mathbb{N} \to \mathbb{R}$ are given by $f(n) = n^2$ and $g(n) = 1 + 2 + \cdots + n$, then $f \asymp g$. TRUE FALSE

2. (5 points) Find a particular solution p_n to the recurrence relation

   $$a_n = -2a_{n-1} + 2a_{n-2} - n \quad (n \geq 2).$$

   (you do not have to worry about initial conditions and you do not need to solve for the homogeneous part q_n)

For this problem, we recall definition 8.2.1 from the textbook, which states:

"Let f and g be functions $\mathbb{N} \to \mathbb{R}$. We say that $f = \mathcal{O}(g)$ if there is an integer n_0 and a positive real number c such that $|f(n)| \leq c|g(n)|$ for all $n \geq n_0$."

3. (6 points) Suppose $f, g : \mathbb{N} \to \mathbb{R}$ are functions whose values $f(n)$ and $g(n)$ are always positive. Use definition 8.2.1 to prove that if $f = \mathcal{O}(g)$, then the function $h(n) = \dfrac{f(n)}{n}$ satisfies $h \prec g$.

   (You cannot use *any* additional theorems or propositions. You must prove $h = \mathcal{O}(g)$ and $g \neq \mathcal{O}(h)$ using definition 8.2.1)

4.