

Quiz 4, Discrete Math (15 points), Fall 2016

The quiz is 20 minutes. Show your work and justify your answers where appropriate. If you write the correct answer without sufficient work or justification, you will receive little or no credit. If you are asked to prove something is true, provide a rigorous mathematical proof to show it is true. Do not attempt proof-by-paragraph.

1. (1 point each) Clearly circle your answer (no justification needed here, and no partial credit given).

(a) When -30 is divided by 7 , the remainder is $r = -2$. TRUE FALSE

(b) If a, b , and c are integers satisfying $a \mid b$ and $a \mid c$, then $a \mid (bc)$. TRUE FALSE

(c) If a, b , and c are integers satisfying $a \mid b$ and $b \mid c$, then $a \mid c$. TRUE FALSE

2. (a) (3 points) Find the binary representation of 25.

$$25 = 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = (11001)_2$$

(b) (2 points) Find the octal representation of 53.

$$53 = 6 \cdot 8^1 + 5 \cdot 8^0 = (65)_8.$$

3. (7 pts) Use the Euclidean Algorithm to find $g = \gcd(95, 41)$, and use your work to find integers m and n so that

$$g = 95m + 41n.$$

(do not just guess the values for m and n !)

Solution:

$$95 = 2 \cdot 41 + 13$$

$$41 = 3 \cdot 13 + 2$$

$$13 = 6 \cdot 2 + \boxed{1}$$

$$2 = 2 \cdot 1 + 0.$$

Thus $g = 1$. To write 1 as a linear combination of 95 and 41, we look at our work and go backwards.

$$1 = 13 - 6 \cdot 2$$

$$1 = 13 - 6(41 - 3 \cdot 13) = -6 \cdot 41 + 19 \cdot 13$$

$$1 = -6 \cdot 41 + 19(95 - 2 \cdot 41) = -6 \cdot 41 + 19 \cdot 95 - 38 \cdot 41$$

$$1 = 19 \cdot 95 - 44 \cdot 41.$$

We found $m = 19$ and $n = -44$.