Quiz 3, Discrete Math (15 points), Fall 2016

The quiz is 20 minutes. Show your work and justify your answers where appropriate. If you write the correct answer without sufficient work or justification, you will receive little or no credit. If you are asked to prove something is true, provide a rigorous mathematical proof to show it is true. Do not attempt proof-by-paragraph.

1. (1 point each) Clearly circle your answer (no justification needed here, and no partial credit given).

   (a) Suppose $\mathcal{R}$ is a binary relation on a set A, and let $a \in A$. If $\mathcal{R}$ is not reflexive, then $(a, a) \notin \mathcal{R}$.
   TRUE FALSE

   (b) Let $A = \{2, 3, 4\}$, and let $\mathcal{R}$ be the relation on A given by $\mathcal{R} = \{(2, 2), (3, 4), (4, 3), (4, 4)\}$. Then $\mathcal{R}$ is transitive. TRUE FALSE

   (c) Let $A = \{1, 2, 3, 4, 5, 6\}$. There is an equivalence relation $\mathcal{R}$ on A that satisfies $\bar{2} \cap \bar{6} = \{4\}$.
   TRUE FALSE

2. (4 points) Suppose $f : A \to B$ and $g : B \to C$ are functions. Prove that if f is one-to-one and g is one-to-one, then $g \circ f$ is one-to-one.

3. (8 points) Define a binary relation $\mathcal{R}$ on $\mathbb{Z}$ in the following manner:
   If $a, b \in \mathbb{Z}$, then $(a, b) \in \mathcal{R}$ if and only if $2a + 5b = 7k$ for some integer k.

   (a) (4 points) Is $\mathcal{R}$ reflexive? Either prove it is reflexive, or give a counterexample showing it is not reflexive.

   (b) (4 points) Prove that $\mathcal{R}$ is transitive.