Quiz 2, Discrete Math (15 points), Fall 2016

Show your work and justify your answers where appropriate. If you write the correct answer without sufficient work or justification, you will receive little or no credit.

1. (3 points) Determine the truth value of

$$\big[p \wedge \big(q \rightarrow ((\neg r) \wedge s)\big)\big] \longleftrightarrow (r \wedge t),$$

where p, q, r, s, and t are all true. Briefly justify your answer.

Solution: False, because the right hand side is true but the left hand side is false, as we show below.

For the $\longleftrightarrow$ statement to be true, both sides must be true or both sides must be false. Since r and t are true, the right side statement $r \wedge t$ is true.

On the left side, $q \rightarrow ((\neg r) \wedge s)$ is false because q is true and $(\neg r) \wedge s$ is false (since $\neg r$ is false). Therefore, the left side statement $p \wedge \big(q \rightarrow ((\neg r) \wedge s)\big)$ is false.

2. (1 point each) Clearly circle your answer (no justification needed here, and no partial credit given).

(a) If A is a set, then $\{A\} \subseteq \mathcal{P}(A)$. **[TRUE]** FALSE
($A \in \mathcal{P}(A)$, so $\{A\} \subseteq \mathcal{P}(A)$)

(b) There is a set X which satisfies $X \subseteq \{X\}$. **[TRUE]** FALSE
($\emptyset \subseteq \{\emptyset\}$)

(c) If A, B, and C are sets, then $A \cup (B \cap C) = (A \cap B) \cup (A \cap C)$. TRUE **[FALSE]**

3. (3 points) Is the statement below true or false? If the statement is true, prove it. If the statement is false, give a counterexample.
If A, B, and C are sets satisfying $A \subseteq (B \cup C)$ and $A \nsubseteq B$, then $A \subseteq C$.

Solution: False. Let $A = \{1, 2\}$, $B = \{1\}$, and $C = \{2\}$. Then $A \subseteq (B \cup C)$ and $A \nsubseteq B$, but $A \nsubseteq C$.

4. (6 points) Prove the statement below. Your steps must be clear, and your argument must be mathematically rigorous (do not just write a paragraph or paragraphs explaining in words why it must be true).

   If A, B, and C are sets, then $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$.

   Solution: Let A, B, and C be sets. We show inclusion both ways.

   ($\subseteq$) Let $x \in A \setminus (B \cap C)$. Then $x \in A$ and $x \notin (B \cap C)$, hence $x \notin B$ or $x \notin C$.

   Case (I): Suppose $x \notin B$. Since $x \in A$, we have $x \in A \setminus B$, hence $x \in (A \setminus B) \cup (A \setminus C)$.

   Case (II): Suppose $x \notin C$. Since $x \in A$, we have $x \in A \setminus C$, hence $x \in (A \setminus B) \cup (A \setminus C)$.

   This proves that $A \setminus (B \cap C) \subseteq (A \setminus B) \cup (A \setminus C)$.

   ($\supseteq$): Let $y \in (A \setminus B) \cup (A \setminus C)$. Then $y \in A \setminus B$ or $y \in A \setminus C$.

   Case (I): Suppose $y \in A \setminus B$. Then $y \in A$ and $y \notin B$, so $y \notin B \cap C$, hence $y \in A \setminus (B \cap C)$.

   Case (II): Suppose $y \in A \setminus C$. Then $y \in A$ and $y \notin C$, so $y \notin B \cap C$, hence $y \in A \setminus (B \cap C)$.

   Therefore, $A \setminus (B \cap C) \supseteq (A \setminus B) \cup (A \setminus C)$.

   We conclude that $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$.