

Quiz 11, Discrete Math (15 points), Fall 2016

The quiz is 20 minutes. Show your work and justify your answers where appropriate. If you write the correct answer without sufficient work or justification, you will receive little or no credit.

1. (a) (1 point) True or false (no partial credit, no justification required):
 K_9 has an Eulerian circuit. TRUE FALSE
(K_9 is connected and every vertex has degree 8)
- (b) (1 point) True or false (no partial credit, no justification required):
A trail is a walk with no repeated vertices. TRUE FALSE
(A trail has no repeated edges; a trail can repeat vertices)
- (c) (1 point) True or false (no partial credit, no justification required):
There is a graph G with 5 vertices, whose degree sequence is 3, 3, 3, 1, 1. TRUE FALSE
(The sum of the degrees would be odd, which is impossible)
- (d) (1 point) True or false (no partial credit, no justification required):
There is a graph G with 5 vertices, whose degree sequence is 4, 4, 3, 2, 1. TRUE FALSE
(The two vertices of degree 4 would be adjacent to all others, thus every vertex must have degree ≥ 2)

2. (3 points) Complete the following definition (be mathematically precise!):
“Let $G_1 = G_1(V_1, \mathcal{E}_1)$ and $G_2 = G_2(V_2, \mathcal{E}_2)$ be graphs. We say that G_1 is isomorphic to G_2 , and write $G_1 \cong G_2$, provided...”

There exists a bijection $\phi : V_1 \rightarrow V_2$ such that:

1. If $vw \in \mathcal{E}_1$ then $\phi(v)\phi(w) \in \mathcal{E}_2$, and
 2. Every edge in \mathcal{E}_2 has the form $\phi(v)\phi(w)$ for some edge $vw \in \mathcal{E}_1$.
3. (4 points) A graph G has 50 edges, four vertices of degree 2, six of degree 5, eight of degree 4, and the rest of degree 6. How many vertices of G have degree 6?

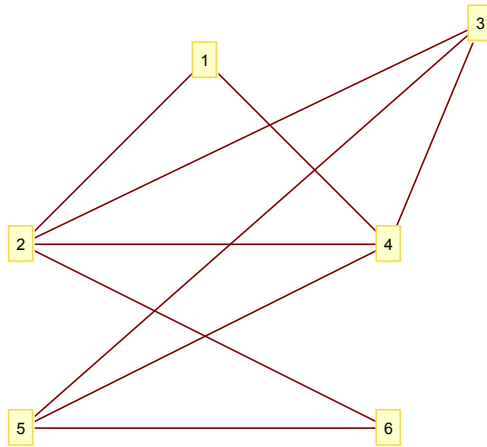
Solution: We use the fact that $\sum_{v \in V} \deg(v) = 2|\mathcal{E}|$. Let r be the number of vertices of degree 6. Then

$$4 \cdot 2 + 6 \cdot 5 + 8 \cdot 4 + 6r = 2 \cdot 50$$

$$70 + 6r = 100 \quad 6r = 30 \quad r = 5.$$

So, there are 5 vertices with degree 6.

4. (4 points) Consider the graph G below, whose vertices are labeled 1 through 6. Does G have an Eulerian trail? If your answer is no, justify your answer. If your answer is yes, justify your answer, and write vertices v and w so that G has an Eulerian trail from v to w .



The degrees of the vertices 1 through 6 are (in order)

2, 4, 3, 4, 3, 2.

Since G is (obviously) connected and it has exactly two vertices with odd degree, it has an Eulerian trail between the two vertices of odd degree, namely vertex 3 and vertex 5.