Math 1553 Worksheet §3.4-3.6

1. True or false. Answer true if the statement is *always* true. Otherwise, answer false. If your answer is false, either give an example that shows it is false or (in the case of an incorrect formula) state the correct formula.

   a) If A is an $n \times n$ matrix and the equation $Ax = b$ has at least one solution for each b in $\mathbf{R}^n$, then A must be invertible.

   b) If A is a 3×4 matrix and B is a 4×2 matrix, then the linear transformation Z defined by $Z(x) = ABx$ has domain $\mathbf{R}^3$ and codomain $\mathbf{R}^2$.

2. A is $m \times n$ matrix, B is $n \times m$ matrix. Select all correct answers from the box. It is possible to have more than one correct answer.

a) Suppose x is in $\mathbf{R}^m$. Then ABx *must be* in:

| Col(A), | Nul(A), | Col(B), | Nul(B) |

b) Suppose x in $\mathbf{R}^n$. Then BAx *must be* in:

| Col(A), | Nul(A), | Col(B), | Nul(B) |

c) If $m > n$, then columns of AB could be linearly $independent, \quad dependent$

3. Consider the following linear transformations:

$$T: \mathbf{R}^3 \longrightarrow \mathbf{R}^2 \quad T \text{ projects onto the } xy\text{-plane, forgetting the } z\text{-coordinate}$$
$$U: \mathbf{R}^2 \longrightarrow \mathbf{R}^2 \quad U \text{ rotates clockwise by } 90°$$
$$V: \mathbf{R}^2 \longrightarrow \mathbf{R}^2 \quad V \text{ scales the } x\text{-direction by a factor of 2.}$$

Let A, B, C be the matrices for T, U, V, respectively.

a) Write A, B, and C.

b) Compute the matrix for $U \circ V \circ T$.

c) Describe U^{-1} and V^{-1}, and compute their matrices.