

**Math 1553 Worksheet §3.2, 3.3**

**1.** Which of the following statements are true? Justify your answer.

**a)** Let  $A$  be a  $3 \times 3$  matrix, such that  $Ax = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$  has a unique solution. Then,

$Ax = \begin{pmatrix} 0 \\ -1 \\ 0 \end{pmatrix}$  also has a unique solution.

**b)** Let  $A$  be a  $3 \times 4$  matrix. Then, the transformation whose standard matrix is  $A$  cannot be onto.

2. Which of the following transformations  $T$  are onto? Which are one-to-one? If the transformation is not onto, find a vector not in the range. If the transformation is not one-to-one, find two vectors with the same image.

a) Counterclockwise rotation by  $32^\circ$  in  $\mathbf{R}^2$ .

b) The transformation  $T : \mathbf{R}^3 \rightarrow \mathbf{R}^2$  defined by  $T(x, y, z) = (z, x)$ .

c) The matrix transformation with standard matrix  $A = \begin{pmatrix} 1 & 6 \\ -1 & 2 \\ 2 & -1 \end{pmatrix}$ .

3. The second little pig has decided to build his house out of sticks. The big bad wolf finds the pig's house and blows it down so that the house is rotated by an angle of  $45^\circ$  in a counterclockwise direction about the  $z$ -axis (look downward onto the  $xy$ -plane the way we usually picture the plane as  $\mathbf{R}^2$ ), and then projected onto the  $xy$ -plane. Find the standard matrix  $A$  for the transformation  $T$  caused by the wolf.