Math 1553 Worksheet §1.2, intro to §1.3

1. **a)** Circle the 'operations' that are legal to use in row reduction, in other words, the operations that will not change the solution set of an arbitrary linear system.

(1) $R_2 = R_3 + 4R_2$

(2) $R_1 = R_2 - R_3$

(3) $R_2 = R_2 + (R_1)^5$

(4) $R_3 = R_3 - \ln(R_2)$

b) These are row operations only. Try performing a column operation: for example, try doubling any column in $\left(\begin{array}{c|c} 1 & 1 \end{array}\right)$. What happens to the solution set?

2. **a)** Which of the following matrices are in row echelon form (REF)? Which are in reduced row echelon form (RREF)?

b) For the matrices that are in REF or RREF, which entries are the pivots? What are the pivot columns?

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right) \qquad \left(\begin{array}{cccc|c} 1 & 1 & 3 & 1 & 1 \\ 0 & 0 & 4 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{array}\right) \qquad \left(\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 0 & 0 & 2 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

c) How many nonzero entries are there in a pivot column of a matrix in RREF?

3. Each matrix below is in RREF. In each case, determine whether the corresponding system of linear equations is consistent, and if so, how many solutions does it have?

(a) $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$, (b) $\left(\begin{array}{cccc|c} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 7 \end{array}\right)$, (c) $\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

4. Write the augmented matrix for the corresponding system of equations and put it in reduced row echelon form. How many solutions does the system have?

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 1 \\ -4x_1 - 9x_2 + 2x_3 &= -1 \\ -3x_2 - 6x_3 &= -3. \end{aligned}$$