# Math 1553 Exam 1, SOLUTIONS, Spring 2026, Ver. B

| **Name** | | **GT ID** | |
|---|---|---|---|

Circle your instructor and lecture below. Be sure to circle the correct choice!

Jankowski (A+HP, 8:25 AM) Jankowski (C, 9:00 AM) Callis (E, 10:00 AM)

Hao (F, 11:00 AM) Poudel (L, 4:00 PM) Van Why (M, 5:00 PM)

Poudel (S, 5:00 PM)

Please read the following instructions carefully.

- Write your initials at the top of each page. The maximum score on this exam is 70 points, and you have 75 minutes to complete it. Each problem is worth 10 points.
- Calculators and cell phones are not allowed. Aids of any kind (notes, text, etc.) are not allowed. If you use pen, you must use black ink.
- As always, RREF means "reduced row echelon form." The "zero vector" in $\mathbf{R}^n$ is the vector in $\mathbf{R}^n$ whose entries are all zero.
- On free response problems, show your work, unless instructed otherwise. A correct answer without appropriate work may receive little or no credit!
- We will hand out loose scrap paper, but it **will not be graded** under any circumstances. All answers and work must be written on the exam itself, with no exceptions.
- This exam is double-sided. You should have enough space to do every problem on the exam, but if you need extra space, you may use the *back side of the very last page of the exam*. If you do this, you must clearly indicate it.
- You may cite any theorem proved in class or in the sections we covered in the text.
- For questions with bubbles, either fill in the bubble completely or leave it blank. **Do not** mark any bubble with "X" or "/" or any such intermediate marking. Anything other than a blank or filled bubble may result in a 0 on the problem, and regrade requests may be rejected without consideration.

*I, the undersigned, hereby affirm that I will not share the contents of this exam with anyone. Furthermore, I have not received inappropriate assistance in the midst of nor prior to taking this exam. I will not discuss this exam with anyone in any form until after 7:45 PM on Wednesday, February 11.*

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1. TRUE or FALSE. Clearly fill in the bubble for your answer. If the statement is *ever* false, fill in the bubble for False. You do not need to show any work, and there is no partial credit. Each question is worth 2 points.

   (a) If the bottom row of an augmented matrix in RREF is $\left(\begin{array}{ccc|c} 0 & 1 & -4 & 1 \end{array}\right)$, then the corresponding system of linear equations must have infinitely many solutions.

   ● True

   ○ False

   (b) If a consistent system of linear equations has more equations than variables, then the system must have exactly one solution.

   ○ True

   ● False

   (c) If $v_1$ and $v_2$ are vectors in $\mathbf{R}^3$, then the vector $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ must be a linear combination of $v_1$ and $v_2$.

   ● True

   ○ False

   (d) Suppose $A$ is a matrix with three columns: $v_1$, $v_2$, and $v_3$. If $b$ is a vector and the equation $Ax = b$ is inconsistent, then $b$ is not in Span $\{v_1, v_2, v_3\}$.

   ● True

   ○ False

   (e) Suppose $A$ is a $2 \times 3$ matrix with 2 pivots, and that $b$ is a vector with the property that $Ax = b$ is consistent. Then the solution set to $Ax = b$ must be a plane.

   ○ True

   ● False

**Problem 1 Solution**.

(a) True: the statement tells us that the second column contains the rightmost pivot of the augmented matrix. Therefore, the system is consistent (since the right column has no pivot), and it has infinitely solutions since the third variable does not have a pivot in its column.

(b) False. For example, the following system has 3 equations and only 2 variables, but it has infinitely many solutions:

$$x - y = 0$$

$$2x - 2y = 0$$

$$3x - 3y = 0.$$

(c) True: the linear combination $0v_1 + 0v_2$ is equal to $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

(d) True. This follows directly from the definition of the term span. The fact that $Ax = b$ is inconsistent means that $b$ is not a linear combination of the columns of $A$, which is precisely to say that $b$ is not a linear combination of $v_1$, $v_2$, and $v_3$.

(e) False. Since $A$ has two pivots but three columns, there is exactly one column of $A$ without a pivot. In other words, there is exactly one free variable in the solution set to $Ax = b$, so the solution set is a line.

2. On this page, you do not need to show work, and only your answers are graded. Parts (a) through (d) are unrelated.

(a) (3 points) Which of the following matrices are in RREF? Fill in the bubble for all that apply.

● $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$

● $\left(\begin{array}{cccc|c} 1 & 5 & -3 & 0 & 9 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$

○ $\left(\begin{array}{cccc|c} 0 & 1 & -5 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right)$

(b) (2 points) Find all real $h$ (if there are any) so that the following linear system of equations has exactly one solution.

$$2x + hy = h$$

$$8x + 24y = -2.$$

○ $h = 0$ only ○ $h = -3$ only ○ $h = 3$ only ○ $h = 6$ only

○ $h = 12$ only ○ $h = -12$ only ○ all $h$ except $-3$

○ all $h$ except 12 ● all $h$ except 6 ○ all $h$ except $-6$

○ all real numbers $h$ ○ the system never has exactly one solution

(c) (2 points) Which **one** vector below is a linear combination of $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix}$?

○ $\begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$ ○ $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ ○ $\begin{pmatrix} 3 \\ 3 \\ 0 \end{pmatrix}$ ● $\begin{pmatrix} 4 \\ 0 \\ 4 \end{pmatrix}$

(d) (3 points) Suppose $v_1$, $v_2$, and $v_3$ are vectors in $\mathbf{R}^2$. Which of the following statements are true? Fill in the bubble for all that apply.

● The vector equation $x_1 v_1 + x_2 v_2 + x_3 v_3 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ must have infinitely many solutions.

○ If $v_1$ is not the zero vector, then Span $\{v_1\}$ consists of exactly one vector.

● If the solution set to $x_1 v_1 + x_2 v_2 + x_3 v_3 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ is a line, then the solution set to the homogeneous equation $x_1 v_1 + x_2 v_2 + x_3 v_3 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ must also be a line.

**Problem 2 Solution**.

(a) (i) and (ii) are in RREF. They meet all the conditions.

(iii) is not in RREF. Its 2nd row has a pivot with a nonzero entry above it.

(b) We row-reduce:

$$\left(\begin{array}{cc|c} 2 & h & h \\ 8 & 24 & -2 \end{array}\right) \xrightarrow{R_2=R_2-4R_1} \left(\begin{array}{cc|c} 2 & h & h \\ 0 & 24-4h & -2-4h \end{array}\right).$$

The system will have exactly one solution precisely when there is a pivot in the spot of $24-4h$, so $24-4h \neq 0$. In other words, $h \neq 6$.

(c) Note that $\begin{pmatrix}1\\2\\1\end{pmatrix}$ and $\begin{pmatrix}2\\-3\\2\end{pmatrix}$ have their first entry equal to their third entry, so any linear combination of them must also have this property. The only choice whose first entry equals its third entry is $\begin{pmatrix}4\\0\\4\end{pmatrix}$, so it must be the answer. We could also check that $\left(\begin{array}{cc|c} 1 & 2 & 4 \\ 2 & -3 & 0 \\ 1 & 2 & 4 \end{array}\right)$ is consistent to confirm it is correct.

(d) (i) is true: the corresponding augmented matrix has three columns to the left of the vertical bar but at most two pivots, so there is at least one free variable in the solution set to the homogeneous system.

(ii) is false: If $v_1$ is not the zero vector, then Span $\{v_1\}$ is a line.

(iii) is true: by the Key Observation in section 2.4, if the solution set to $x_1v_1 + x_2v_2 + x_3v_3 = \begin{pmatrix}1\\-1\end{pmatrix}$ is a line, then the corresponding solution set to the homogeneous equation $x_1v_1 + x_2v_2 + x_3v_3 = \begin{pmatrix}0\\0\end{pmatrix}$ will be a parallel line through the origin. Alternatively, we could see that this statement is true just by counting pivots.

3. On this page, you do not need to show work, and only your answers are graded. Parts (a) through (d) are unrelated.

(a) (2 points) Solve the vector equation $x_1 \begin{pmatrix} -1 \\ 3 \end{pmatrix} + x_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -5 \\ 1 \end{pmatrix}$.

Fill in the bubble for your answer below.

○ $x_1 = 1$ and $x_2 = 2$ ○ $x_1 = -1$ and $x_2 = 2$ ● $x_1 = 1$ and $x_2 = -2$

○ $x_1 = -1$ and $x_2 = 1$ ○ $x_1 = \frac{7}{5}$ and $x_2 = -\frac{16}{5}$

○ $x_1 = -1$, $x_2 = -2$ ○ there are no solutions ○ none of these

(b) (3 points) Suppose we are given a system of linear equations and its corresponding augmented matrix. Which of the following statements must be true? Fill in the bubble for all that apply.

● If the rightmost column of the augmented matrix has a 0 for every entry, then the system must be consistent.

● If the augmented matrix has a pivot in every column, then the system must be inconsistent.

○ If the augmented matrix has a pivot in every row, then the system must be consistent.

(c) (2 points) Find all real values of $h$ so that Span $\left\{ \begin{pmatrix} 6 \\ -1 \end{pmatrix}, \begin{pmatrix} h \\ 3 \end{pmatrix} \right\} = \mathbf{R}^2$.

Fill in the bubble for your answer below.

○ $h = -1$ only ○ $h = 1$ only ○ $h = -18$ only ○ $h = 18$ only

○ $h = 2$ only ○ $h = -2$ only ● All $h$ except $-18$

○ All $h$ except 18 ○ All $h$ except 2 ○ All $h$ except $-2$

○ none of these

(d) (3 points) Suppose we are given a **consistent** linear system of 5 equations in 6 variables that is represented by a matrix equation $Ax = b$. Answer the following.

i. If $x$ is a solution to the matrix equation, then $x$ is a vector in...

○ $\mathbf{R}^3$ ○ $\mathbf{R}^4$ ○ $\mathbf{R}^5$ ● $\mathbf{R}^6$ ○ $\mathbf{R}^7$

ii. The vector $b$ is in...

○ $\mathbf{R}^3$ ○ $\mathbf{R}^4$ ● $\mathbf{R}^5$ ○ $\mathbf{R}^6$ ○ $\mathbf{R}^7$

iii. How many solutions does this consistent system have?

○ exactly one ● infinitely many ○ not enough info to determine

**Problem 3 Solution**.

(a) We row-reduce:

$$\left(\begin{array}{cc|c} -1 & 2 & -5 \\ 3 & 1 & 1 \end{array}\right) \xrightarrow{R_2=R_1+3R_2} \left(\begin{array}{cc|c} -1 & 2 & -5 \\ 0 & 7 & -14 \end{array}\right) \xrightarrow[R_1=-R_1]{R_2=R_2/7} \left(\begin{array}{cc|c} 1 & -2 & 5 \\ 0 & 1 & -2 \end{array}\right)$$

$$\xrightarrow{R_1=R_1+2R_2} \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -2 \end{array}\right), \quad \text{so } x_1 = 1 \text{ and } x_2 = -2.$$

(b) (i) is true: if the rightmost column of the augmented matrix is all zeros, then the system is homogeneous and therefore must be consistent. Alternatively, we could just argue using pivots. If the rightmost column is all zeros, then it cannot have a pivot, so the system is consistent.

(ii) is true: this means that the rightmost column has a pivot, so the system is inconsistent. Just as an illustration:

$$\left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right).$$

(iii) is not necessarily true, for example the following augmented matrix has a pivot in every row but represents an inconsistent system.

$$\left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 0 & 1 \end{array}\right).$$

(c) We look for when the matrix below has a pivot in every row:

$$\begin{pmatrix} 6 & h \\ -1 & 3 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} -1 & 3 \\ 6 & h \end{pmatrix} \xrightarrow{R_2=R_2+6R_1} \begin{pmatrix} -1 & 3 \\ 0 & h+18 \end{pmatrix}.$$

The matrix has a pivot in every row as long as $h + 18 \neq 0$, so $h \neq -18$.

(d) To have 5 equations and 6 variables means that there are four rows and five columns to the *left* of the augment bar, so $A$ is a $5 \times 6$ matrix. This means that $x$ is a vector in $\mathbf{R}^6$ and $b$ is in $\mathbf{R}^5$. Since there are more variables than equations in this consistent linear system, there must be infinitely many solutions.

Another way to approach the problem is to note that since there are 6 variables, any solution $x$ has 6 entries. By definition of multiplication, this means $A$ must have 6 columns. Similarly, the fact that there are 5 equations means $b$ has 5 entries if you look at the shape of the augmented matrix.

4. On this page, you do not need to show work. Only your answers are graded. Parts (a) through (d) are unrelated.

(a) (2 points) Which **one** of the following equations is **not** a linear equation in $x$, $y$, and $z$? Fill in the bubble for your answer

● $3x - 2yz = 1$ ○ $2x - y - z = 4$ ○ $\ln(2)x - \sin(\pi/7)y + z = 2$

(b) (3 points) In the space provided, write three **different** vectors $v_1$, $v_2$, $v_3$ in $\mathbf{R}^3$ that satisfy both of the following properties:

- Span$\{v_1, v_2, v_3\}$ is a plane.
- $v_3$ is **not** a linear combination of $v_1$ and $v_2$.

**Solution**: We choose vectors $v_1$ and $v_2$ so that $v_1 \neq v_2$ and one is a scalar multiple of the other, then choose $v_3$ to be some vector that is not in the span of $v_1$ and $v_2$. In particular, $v_3$ **cannot** be the zero vector, since that is in the span of every set of vectors. One correct answer is $v_1 = \begin{pmatrix}1\\0\\0\end{pmatrix}$, $v_2 = \begin{pmatrix}2\\0\\0\end{pmatrix}$, $v_3 = \begin{pmatrix}0\\1\\0\end{pmatrix}$.

(c) (2 points) Compute $\begin{pmatrix}1 & -3 & 1\\0 & 2 & 4\end{pmatrix}\begin{pmatrix}0\\-1\\5\end{pmatrix}$. Fill in the bubble for your answer.

○ $\begin{pmatrix}8\\22\end{pmatrix}$ ○ $\begin{pmatrix}0 & 3 & 5\\0 & -2 & 20\end{pmatrix}$ ● $\begin{pmatrix}8\\18\end{pmatrix}$ ○ $\begin{pmatrix}-1 & 3 & -1\\0 & 10 & 20\end{pmatrix}$ ○ $\begin{pmatrix}3\\22\end{pmatrix}$

(d) (3 pts) Consider a consistent matrix equation $Ax = b$ whose solution set is:

$$\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} = \begin{pmatrix}2\\0\\-4\end{pmatrix} + x_2\begin{pmatrix}-3\\1\\0\end{pmatrix}. \quad \text{(where } x_2 \text{ is a free variable)}$$

(i) Which vectors below are solutions to $Ax = b$? Fill in the bubble for all that apply.

○ $\begin{pmatrix}-3\\1\\0\end{pmatrix}$ ● $\begin{pmatrix}2\\0\\-4\end{pmatrix}$ ○ $\begin{pmatrix}-6\\2\\0\end{pmatrix}$

(ii) Which one of the following describes the solution set to the corresponding **homogeneous** matrix equation $Ax = 0$?

○ The line through $\begin{pmatrix}0\\0\\0\end{pmatrix}$ and $\begin{pmatrix}2\\0\\-4\end{pmatrix}$. ○ Span $\left\{\begin{pmatrix}2\\0\\-4\end{pmatrix}, \begin{pmatrix}-3\\1\\0\end{pmatrix}\right\}$.

● The line through $\begin{pmatrix}0\\0\\0\end{pmatrix}$ and $\begin{pmatrix}-3\\1\\0\end{pmatrix}$. ○ The line through $\begin{pmatrix}0\\0\\0\end{pmatrix}$ and $\begin{pmatrix}2\\0\\-4\end{pmatrix}$.

**Problem 4 Solution**.

(a) The $2yz$ term in the second equation makes it non-linear. The third equation might look non-linear at a glance because it has natural log and sine, but the variables are not inside those functions.

(b) Many examples possible, for example

$$v_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \qquad v_2 = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}, \qquad v_3 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}.$$

We can just take $v_1$ and $v_2$ to be different vectors that are scalar multiples of each other, then take $v_3$ to be a vector that is not in their span.

(c) $\begin{pmatrix} 1 & -3 & 1 \\ 0 & 2 & 4 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \\ 5 \end{pmatrix} = 0 \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} -3 \\ 2 \end{pmatrix} + 5 \begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} + \begin{pmatrix} 5 \\ 20 \end{pmatrix} = \begin{pmatrix} 8 \\ 18 \end{pmatrix}.$

(d) (i): From the Key Observation of section 2.4: the sol. set is given by taking the particular solution $\begin{pmatrix} 2 \\ 0 \\ -4 \end{pmatrix}$ and adding all homogeneous solutions, which are all scalar multiples of $\begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}$.

- $\begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}$ is a solution to the **homogeneous** matrix equation $Ax = 0$.
- $\begin{pmatrix} 2 \\ 0 \\ -4 \end{pmatrix}$ is a solution to $Ax = b$.
- $\begin{pmatrix} -6 \\ 2 \\ 0 \end{pmatrix}$ is a solution to the **homogeneous** equation $Ax = 0$; it is $2 \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}$.

(ii) The homogeneous solution set is all scalar multiples of $\begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}$. Geometrically, these form the line that goes through the origin and $\begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}$.

5. Free response. Show your work unless otherwise indicated! A correct answer without sufficient work may receive little or no credit.
For this page of the exam, consider the following linear system of equations in the variables $x_1,\ x_2,\ x_3$:

$$
\begin{aligned}
x_1 - 2x_2 + 5x_3 &= 10 \\
3x_1 + 2x_2 - x_3 &= 6 \\
-4x_1 + x_2 - 6x_3 &= -19 \\
2x_1 - 4x_2 + 10x_3 &= 20.
\end{aligned}
$$

(a) (5 points) Write the system in the form of an augmented matrix, and put the augmented matrix in reduced row echelon form.

**Solution**: We box the pivots below as a guide.

$$
\left(\begin{array}{ccc|c} \boxed{1} & -2 & 5 & 10 \\ 3 & 2 & -1 & 6 \\ -4 & 1 & -6 & -19 \\ 2 & -4 & 10 & 20 \end{array}\right)
\xrightarrow[R_3=R_3+4R_1,\ R_4=R_4-2R_1]{R_2=R_2-3R_1}
\left(\begin{array}{ccc|c} \boxed{1} & -2 & 5 & 10 \\ 0 & \boxed{8} & -16 & -24 \\ 0 & -7 & 14 & 21 \\ 0 & 0 & 0 & 0 \end{array}\right)
$$

$$
\xrightarrow[\text{then } R_3=R_3+7R_2]{R_2=R_2/8}
\left(\begin{array}{ccc|c} \boxed{1} & -2 & 5 & 10 \\ 0 & \boxed{1} & -2 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)
$$

$$
\xrightarrow{R_1=R_1+2R_2}
\left(\begin{array}{ccc|c} \boxed{1} & 0 & 1 & 4 \\ 0 & \boxed{1} & -2 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)
$$

(b) (4 pts) The system is consistent. Write its solution set in parametric **vector** form.

**Solution:** We see $x_1 + x_3 = 4$ and $x_2 - 2x_3 = -3$, so:

$$x_1 = 4 - x_3, \qquad x_2 = -3 + 2x_3, \qquad x_3 = x_3\ (x_3 \text{ real}).$$

$$
\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 - x_3 \\ -3 + 2x_3 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix} + \begin{pmatrix} -x_3 \\ 2x_3 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}
$$

(c) (1 point) Write **one** vector $x$ that solves the linear system of equations. There is no partial credit on this part, so take time to check by hand that your answer is correct, and if it is not correct then go back and check your work above!

**Solution**: One solution to the system is $\begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix}$, and to get more solutions to the system we can add any multiple of the **homogeneous** solution $\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$, for example:

$$\begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix},$$

$$\begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix} + 2\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix},$$

$$\begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ -5 \\ -1 \end{pmatrix},$$

etc.

6. Free response. Show your work unless otherwise indicated! A correct answer without appropriate work will receive little or no credit. Parts (a) and (b) are unrelated.

(a) (5 points) Consider the linear system below, where $h$ and $k$ are real numbers.

$$6x - hy = -5$$

$$x + 3y = k.$$

Find all values of $h$ and $k$ (if there are any) so that the system is inconsistent. Write your answer in the space provided below.

$$\boxed{h = -18 \quad \text{and} \quad k \neq -\frac{5}{6}}$$

**Solution**: We form an augmented matrix and row-reduce:

$$\left(\begin{array}{cc|c} 6 & -h & -5 \\ 1 & 3 & k \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_2} \left(\begin{array}{cc|c} 1 & 3 & k \\ 6 & -h & -5 \end{array}\right) \xrightarrow{R_2 = R_2 - 6R_1} \left(\begin{array}{cc|c} 1 & 3 & k \\ 0 & h+18 & -5-6k \end{array}\right).$$

For the system to be inconsistent, the second row must have a pivot in its right-most column. Therefore,

- $h + 18 = 0$, so $h = -18$
- $-5 - 6k \neq 0$, so $6k \neq -5$, thus $k \neq -5/6$.

(b) (5 points) Find the solution set for the vector equation

$$x_1 \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix} + x_2 \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + x_3 \begin{pmatrix} 1 \\ -1 \\ 10 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 16 \end{pmatrix}.$$

Write your answer in parametric form in the box below.

**Solution**: We row-reduce.

$$\left(\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & -1 & -1 \\ 3 & -1 & 10 & 16 \end{array}\right) \xrightarrow{R_3 = R_3 - 3R_1} \left(\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & -1 & -1 \\ 0 & -7 & 7 & 7 \end{array}\right) \xrightarrow[R_1 = R_1 - 2R_2]{R_3 = R_3 + 7R_2} \left(\begin{array}{ccc|c} 1 & 0 & 3 & 5 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right).$$

This gives us $x_1 + 3x_3 = 5$, $x_2 - x_3 = -1$, and $x_3$ free, so our parametric form is:

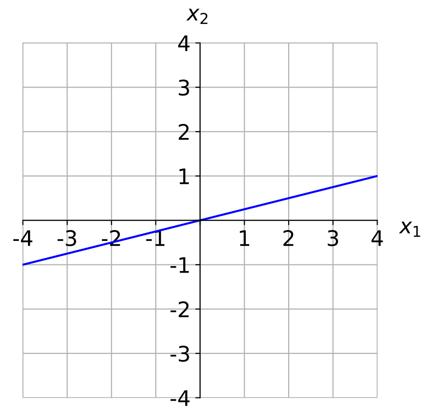
$$\boxed{x_1 = 5 - 3x_3, \qquad x_2 = -1 + x_3, \qquad x_3 = x_3}.$$

7. Free response. Show your work unless otherwise indicated! A correct answer without sufficient work will receive little or no credit. Parts (a) and (b) are unrelated.

(a) (6 points) Let $A = \begin{pmatrix} 1 & -4 \\ -2 & 8 \end{pmatrix}$.

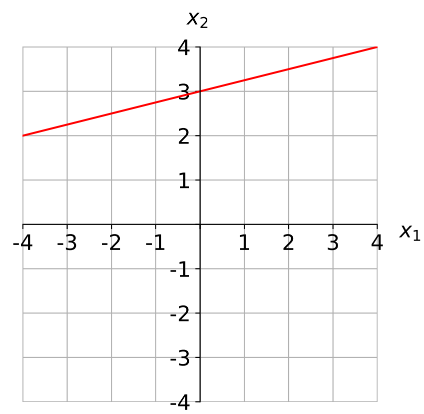
i. On the graph provided below, draw the solution set to $Ax = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$. Briefly show your work for how you found the solution set.

**Solution**: $\left(\begin{array}{cc|c} 1 & -4 & 0 \\ -2 & 8 & 0 \end{array}\right) \to \left(\begin{array}{cc|c} 1 & -4 & 0 \\ 0 & 0 & 0 \end{array}\right)$, so $x_1 = 4x_2$ and $x_2 = x_2$ where $x_2$ is any real number. This is the span of $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$.




ii. For some $b$, the vector $x = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$ is a solution to $Ax = b$. Draw the solution set to $Ax = b$ below. You do not need to show your work on this part.

**Solution**: By the Key Observation of 2.4, the solution set to $Ax = b$ is the line through $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ parallel to the span of $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$.



(b) (4 points) In the box provided, write an augmented matrix in RREF that corresponds to a system of linear equations in the variables $x_1$, $x_2$, and $x_3$ whose solution set is

$$x_1 = 3 - 4x_3, \qquad x_2 = -1, \qquad x_3 = x_3 \ (x_3 \text{ real}).$$

Briefly justify your answer.

**Solution**: We need 3 columns to the left of the vertical bar, since there are 3 variables. Also, we need $x_3$ to be a free variable.

- The equation $x_1 = 3 - 4x_3$ is equivalent to $x_1 + 4x_3 = 3$ which is the row $\left(\begin{array}{ccc|c} 1 & 0 & 4 & 3 \end{array}\right)$.
- The statement $x_2 = -1$ is the row $\left(\begin{array}{ccc|c} 0 & 1 & 0 & -1 \end{array}\right)$.

One answer is

$$\left(\begin{array}{ccc|c} 1 & 0 & 4 & 3 \\ 0 & 1 & 0 & -1 \end{array}\right).$$

We can get other correct answers by just adding rows of zeros, which do not change anything about the solution set. For example,

$$\left(\begin{array}{ccc|c} 1 & 0 & 4 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right), \qquad \left(\begin{array}{ccc|c} 1 & 0 & 4 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right).$$

**This page is reserved ONLY for work that did not fit elsewhere on the exam.**

**If you use this page, please clearly indicate (on the problem's page and here) which problems you are doing.**