Studypalooza, Spring 2026

There are 50 problems for the Studypalooza event for Reading Day in Spring 2026. They were compiled using PreTeXt with the specific intention of producing a PDF that meets the WCAG 2.0 accessibility guidelines.

1. True or false: If $\{u, v, w\}$ is a set of linearly dependent vectors, then w must be a linear combination of u and v.

2. Find the value of k that makes the following vectors linearly dependent: $\left\{ \begin{pmatrix} -3 \\ 0 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ -3 \\ k \end{pmatrix}, \begin{pmatrix} 3 \\ -1 \\ -1 \end{pmatrix} \right\}$.

3. True or false: If $\{u, v\}$ is a basis for a subspace W, then $\{u - v, u + v\}$ is also a basis for W.

4. Which of the following are subspaces of $\mathbf{R}^4$?

(a) The set $W = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \text{ in } \mathbf{R}^4 \;\middle|\; 2x - y - z = 0 \right\}$

(b) The set of solutions to $\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 3 & 0 & -1 \end{pmatrix} x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

5. True or false: The set W of vectors $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ with $abc = 0$. Then W is closed under addition.

6. Write the matrices for each of the following.

(a) Counterclockwise rotation by 90°.

(b) Reflection across the line $y = x$.

(c) Clockwise rotation by 90°

(d) Reflection across the x-axis.

(e) Reflection across the y-axis.

7. Find all k so that $\begin{pmatrix} 2 \\ k \\ 1 \end{pmatrix}$ and $\begin{pmatrix} k \\ 1 \\ -6 \end{pmatrix}$ are orthogonal.

8. Find all k so that the matrix transformation corresponding to the following matrix is not onto: $\begin{pmatrix} 1 & 3 & 9 \\ 2 & 6 & k \end{pmatrix}$.

9. Let $T : \mathbf{R}^a \to \mathbf{R}^b$ be a transformation. Match the four statements with their corresponding terms listed below.

(a) For each y in $\mathbf{R}^b$, there is at most one x in $\mathbf{R}^a$ so that $T(x) = y$.

(b) For each y in $\mathbf{R}^b$, there is at least one x in $\mathbf{R}^a$ so that $T(x) = y$.

(c) For each y in $\mathbf{R}^b$, there is at exactly one x in $\mathbf{R}^a$ so that $T(x) = y$.

(d) For each x in $\mathbf{R}^a$, there is exactly one y in $\mathbf{R}^b$ so that $T(x) = y$.

The terms are: "transformation," "one-to-one," "onto," and "invertible."

10. True or false: Suppose A is a 4×6 matrix. Then the dimension of the null space of A is at most 2.

11. Complete the entries of A so that $\mathrm{Col}(A) = \mathrm{Span}\left\{\begin{pmatrix}1\\2\end{pmatrix}\right\}$ and $\mathrm{Nul}(A) = \mathrm{Span}\left\{\begin{pmatrix}1\\1\end{pmatrix}\right\}$.

$A = \begin{pmatrix} r & 1 \\ s & 2 \end{pmatrix}.$

12. Suppose $T : \mathbf{R}^7 \to \mathbf{R}^9$ is a linear transformation with standard matrix A, and suppose the range of T has a basis consisting of 3 vectors. What is the nullity of A?

13. Define $T : \mathbf{R}^3 \to \mathbf{R}^4$ by $T(x, y, z) = (0, x - y, y - x, z)$.

Is T one-to-one? Is T onto?

14. Suppose A is a 7×5 matrix and its null space is a line, and let T be the matrix transformation $T(x) = Ax$. What is true about the range of T?

(a) It is a 4-dimensional subspace of $\mathbf{R}^5$.

(b) It is a 6-dimensional subspace of $\mathbf{R}^7$.

(c) It is a 4-dimensional subspace of $\mathbf{R}^7$.

(d) It is a 6-dimensional subspace of $\mathbf{R}^5$.

(e) none of these

15. Say that $S : \mathbf{R}^2 \to \mathbf{R}^3$ and $T : \mathbf{R}^3 \to \mathbf{R}^4$ are linear transformations. Which one of the following must be true about $T \circ S$?

(a) It is one-to-one

(b) It is not one-to-one

(c) It is onto

(d) The composition $T \circ S$ is not defined

(e) It is not onto

16. True or false: Suppose that A is an invertible $n \times n$ matrix. Then $A + A$ must be invertible.

17. True or false: Suppose A is a 3×3 matrix and the equation $Ax = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$ has exactly one solution. Then A must be invertible.

18. Suppose A and B are $n \times n$ matrices and AB is not invertible. Which one of the following must be true?

(a) A is not invertible.

(b) B is not invertible.

(c) At least one of the matrices A and B is not invertible.

(d) none of these.

19. Suppose A and B are 3×3 matrices, with $\det(A) = 3$ and $\det(B) = -6$. Find $\det(2A^{-1}B)$.

20. Let A be the 3×3 matrix satisfying $Ae_1 = e_3$, $Ae_2 = e_2$, and $Ae_3 = 2e_1$ (recall that e_1, e_2, and e_3 are the standard basis vectors of $\mathbf{R}^3$).

Find $\det(A)$.

21. Suppose A is a square matrix and $\lambda = -1$ is an eigenvalue of A. Which one of the following must be true?

(a) $\text{Nul}(A + I) = \{0\}$.

(b) A is invertible.

(c) The columns of A are linearly independent.

(d) For some nonzero vector x, the vectors Ax and x are linearly dependent.

(e) The equation $Ax = x$ has only the trivial solution.

22. Suppose A is a 4×4 matrix with characteristic polynomial $-\lambda(1 - \lambda)^2(5 - \lambda)$.

What is the rank of A?

23. Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation that reflects across the line $x_2 = 2x_1$. Find the value of k so that $T\begin{pmatrix} 2 \\ k \end{pmatrix} = \begin{pmatrix} 2 \\ k \end{pmatrix}$.

24. Find the value of k so that $\begin{pmatrix} 1 & k \\ 1 & 3 \end{pmatrix}$ has one real eigenvalue with algebraic multiplicity 2.

25. Suppose A is a diagonalizable matrix with characteristic polynomial $\det(A - \lambda I) = (1-\lambda)^3(2-\lambda)(3-\lambda)$.

(a) What is the dimension of the 1-eigenspace of A?

(b) For some n, the 1-eigenspace of A is a subspace of $\mathbf{R}^n$. What is n?

26. Find the value of t so that $\lambda = 3$ is an eigenvalue of $\begin{pmatrix} 1 & t & 3 \\ 1 & 1 & 1 \\ 3 & 0 & 3 \end{pmatrix}$.

27. Suppose A is a 2×2 matrix with characteristic polynomial $(1-\lambda)(2-\lambda)$. Find the characteristic polynomial of A^2.

28. Suppose that x is an eigenvector of a matrix A corresponding to $\lambda = 3$ and that x is also an eigenvector of a matrix B corresponding to $\lambda = 4$.

Determine if x an eigenvector for $2A - B$. If so, find the corresponding eigenvalue.

29. Suppose A is a 4×4 matrix with eigenvalues $0, 1,$ and 2, where the eigenvalue 1 has algebraic multiplicity 2. Which of the following statements must be true? Select all that apply.

(a) A is not invertible.

(b) A is not diagonalizable.

30. True or false: Suppose A is a 5×5 matrix with real entries. Then A must have at least one real eigenvalue.

31. Suppose A is a positive stochastic matrix satisfying $A\begin{pmatrix}3/5\\2/5\end{pmatrix} = \begin{pmatrix}3/5\\2/5\end{pmatrix}$, and let $v = \begin{pmatrix}5\\95\end{pmatrix}$.

As n gets very large, what vector does $A^n v$ approach?

32. Suppose A is a 4×4 matrix with rank 2. Which one of the following must be true?

(a) A must have four distinct eigenvalues.

(b) A is not diagonalizable.

(c) A is diagonalizable.

(d) A cannot have four distinct eigenvalues.

33. Suppose A is a 2×2 matrix whose entries are real numbers, and suppose that $\lambda = 1 + i$ is an eigenvalue of A with corresponding eigenvector $\begin{pmatrix} 2 \\ 1+i \end{pmatrix}$. Which one of the following statements must be true?

(a) A must have eigenvalue $1 - i$ with corresponding eigenvector $\begin{pmatrix} 2 \\ 1+i \end{pmatrix}$.

(b) A must have eigenvalue $1 - i$ with corresponding eigenvector $\begin{pmatrix} 2 \\ 1-i \end{pmatrix}$.

(c) A must have eigenvalue $1 + i$ with corresponding eigenvector $\begin{pmatrix} 2 \\ 1-i \end{pmatrix}$.

(d) none of these

34. Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation of rotation 45° clockwise, and let A be the standard matrix for T.

Which one of the following statements is true?

(a) A has two distinct real eigenvalues.

(b) A has one complex eigenvalue with algebraic multiplicity 2.

(c) A has one real eigenvalue with algebraic multiplicity 2.

(d) A has two distinct complex eigenvalues.

35. Suppose u and v are orthogonal unit vectors (recall that "unit" vector means length 1).

Find the dot product $(3u - 8v) \cdot 4u$.

36. Find all values of k so that $\begin{pmatrix} 2 \\ k \\ 1 \end{pmatrix}$ and $\begin{pmatrix} k \\ 1 \\ -6 \end{pmatrix}$ are orthogonal.

37. True or false: If W is a subspace of $\mathbf{R}^{100}$ and v is in $W^\perp$, then the orthogonal projection of v onto W must be the zero vector.

38. True or false: Suppose W is a subspace of $\mathbf{R}^n$. If x is a vector in $\mathbf{R}^n$ and x_W is the orthogonal projection of x onto W, then $x \cdot x_W$ must be 0.

39. Suppose A is an invertible 3×3 matrix. What is the dot product of the second row of A and the third column of AA^{-1}?

(a) 0

(b) 1

(c) -1

(d) 2

(e) -2

(f) not enough info

40. Find the orthogonal projection of $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ onto $\text{Span}\left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$.

41. Find the orthogonal projection of $\begin{pmatrix} 6 \\ 5 \\ 4 \end{pmatrix}$ onto $\mathrm{Span}\left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right\}$.

42. Let B be the standard matrix for the orthogonal projection of $\mathbf{R}^3$ onto

$$W = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ in } \mathbf{R}^3 \;\middle|\; x + y + 2z = 0 \right\}.$$

What is the dimension of the 1-eigenspace of B?

43. Let W be the subspace of $\mathbf{R}^4$ given by all vectors $\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix}$ satisfying $x+y+z+w=0$. Find the dimension of $W^{\perp}$.

44. True or false: If b is a vector in the column space of a matrix A, then every solution to $Ax=b$ is also a least-squares solution to $Ax=b$.

45. True or false: If A is an $m \times n$ matrix, b is in $\mathbf{R}^m$, and $\hat{x}$ is a least-squares solution to $Ax = b$, then $\hat{x}$ is the point in $\mathrm{Col}(A)$ that is closest to b.

46. Find the least-squares solution $\hat{x}$ to $\begin{pmatrix} 6 \\ -2 \\ -2 \end{pmatrix} x = \begin{pmatrix} 14 \\ -2 \\ 0 \end{pmatrix}$.

47. Find the best fit line $y = Mx + B$ for the data points below using least squares: $(-7, -22)$, $(0, -2)$, and $(7, 6)$.

48. Let $A = \begin{pmatrix} 4 & 1 \\ 5 & 2 \end{pmatrix} \begin{pmatrix} -3 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 4 & 1 \\ 5 & 2 \end{pmatrix}^{-1}$. Find $A^3 \begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

49. True or false: If A is a diagonalizable 6×6 matrix, then A has 6 different eigenvalues.

50. Find the eigenvalues of $A = \begin{pmatrix} 1 & 4 \\ 4 & 7 \end{pmatrix}$.