

**Math 1553 Worksheet §5.2 - §5.4**

- 1.** Answer yes, no, or maybe. Justify your answers. In each case,  $A$  is a matrix whose entries are real numbers.

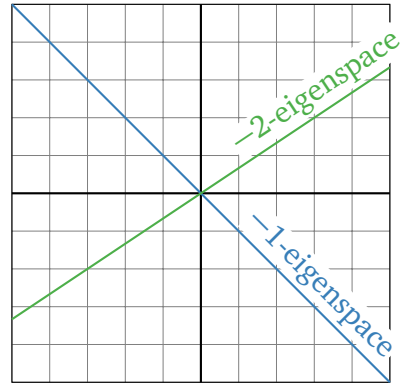
**a)** Suppose  $A = \begin{pmatrix} 3 & 0 & 0 \\ 5 & 1 & 0 \\ -10 & 4 & 7 \end{pmatrix}$ . Then the characteristic polynomial of  $A$  is

$$\det(A - \lambda I) = (3 - \lambda)(1 - \lambda)(7 - \lambda).$$

**b)** If  $A$  is a  $3 \times 3$  matrix with characteristic polynomial  $-\lambda(\lambda - 5)^2$ , then the 5-eigenspace is 2-dimensional.

**c)** If  $A$  is an invertible  $2 \times 2$  matrix, then  $A$  is diagonalizable.

2. The eigenspaces of some  $2 \times 2$  matrix  $A$  are drawn below. Write an invertible matrix  $C$  and a diagonal matrix  $D$  so that  $A = CDC^{-1}$ . Can you find another pair of  $C$  and  $D$  so that  $A = CDC^{-1}$ ?



3. Suppose  $A$  is a  $2 \times 2$  matrix satisfying

$$A \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}, \quad A \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

- a) Diagonalize  $A$  by finding  $2 \times 2$  matrices  $C$  and  $D$  (with  $D$  diagonal) so that  $A = CDC^{-1}$ .

- b) Find  $A^{17}$ .