Math 1553 Worksheet §2.3, S2.4

1. True or false. If the statement is *always* true, answer true. Otherwise, answer false and give an example to show it is false.

   a) Suppose A is an $m \times n$ matrix and b is a vector in $\mathbf{R}^m$. If $Ax = b$ is inconsistent, then A does not have a pivot in every column.

   b) If A is a 4×3 matrix, then the equation $Ax = b$ is inconsistent for some b in $\mathbf{R}^4$.

   c) Suppose A is a 3×3 matrix with two pivots, and suppose that b is a vector so that $Ax = b$ is consistent. Then the solution set for $Ax = b$ is a plane.
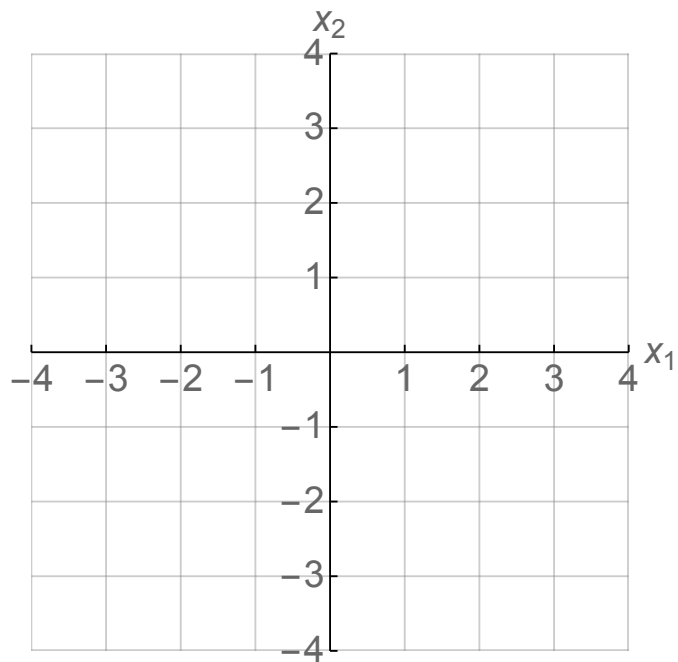
2. Let $A = \begin{pmatrix} 1 & -1 \\ 4 & -4 \end{pmatrix}$. On the same graph, draw each of the following:

(a) The span of the columns of A.

(b) The set of solutions to $Ax = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

(c) The set of solutions to $Ax = \begin{pmatrix} 2 \\ 8 \end{pmatrix}$.

Label each of these clearly.

3. Find the set of solutions to $x_1 - 3x_2 + 5x_3 = 0$ and write your answer in parametric vector form. Next, find the set of solutions to $x_1 - 3x_2 + 5x_3 = 3$ and write the solutions in parametric vector form. How do the solution sets compare geometrically?