

Math 1553 Worksheet §5.4-5.6

1. True or false. Justify your answer.

a) A  $3 \times 3$  matrix  $A$  can have a non-real complex eigenvalue with multiplicity 2.

b) It is possible for a  $2 \times 2$  stochastic matrix to have  $-i/2$  as an eigenvalue.

2. Let  $A = \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}^{-1}$ , and let  $x = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ . What happens to  $A^n x$  as  $n$  gets very large?

3. Let  $A = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$ . Find all eigenvalues of  $A$ . For each eigenvalue, find an associated eigenvector.

4. Axel and Billy are magicians who compete for customers in a group of 180 people. Today, Axel has 120 customers and Billy has 60 customers. Each day:
- 30% of Axel's customers keep attending Axel's show, while 70% of Axel's customers switch to Billy's show.
  - 80% of Billy's customers attend Billy's show, while 20% of Billy's customers switch to Axel's show.
- (a) Write a positive stochastic matrix  $B$  and a vector  $x$  so that  $Bx$  will give the number of customers for Axel's show and Billy's show (in that order) tomorrow. You do not need to compute  $Bx$ .

(b) Find the steady-state vector  $w$  for  $B$ .

(c) In the long run, roughly how many daily customers will Billy have?