Math 1553 Exam 1, SOLUTIONS, Spring 2025

Name		**GT ID**	

Circle your instructor and lecture below. Be sure to circle the correct choice!

Jankowski (A+HP, 8:25-9:15 AM)   Jankowski (C, 9:30-10:20 AM)

Al Ahmadieh (I, 2:00-2:50 PM)   Al Ahmadieh (M, 3:30-4:20 PM)

Please read the following instructions carefully.

- Write your initials at the top of each page. The maximum score on this exam is 70 points, and you have 75 minutes to complete it. Each problem is worth 10 points.
- There are no calculators or aids of any kind (notes, text, etc.) allowed.
- As always, RREF means "reduced row echelon form." The "zero vector" in $\mathbf{R}^n$ is the vector in $\mathbf{R}^n$ whose entries are all zero.
- On free response problems, show your work, unless instructed otherwise. A correct answer without appropriate work may receive little or no credit!
- We will hand out loose scrap paper, but it **will not be graded** under any circumstances. All answers and work must be written on the exam itself, with no exceptions.
- This exam is double-sided. You should have more than enough space to do every problem on the exam, but if you need extra space, you may use the *back side of the very last page of the exam.* If you do this, you must clearly indicate it.
- You may cite any theorem proved in class or in the sections we covered in the text.
- For questions with bubbles, either fill in the bubble completely or leave it blank. **Do not** mark any bubble with "X" or "/" or any such intermediate marking. Anything other than a blank or filled bubble may result in a 0 on the problem, and regrade requests may be rejected without consideration.

I, the undersigned, hereby affirm that I will not share the contents of this exam with anyone. Furthermore, I have not received inappropriate assistance in the midst of nor prior to taking this exam. I will not discuss this exam with anyone in any form until after 7:45 PM on Wednesday, February 5.

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1. TRUE or FALSE. Clearly fill in the bubble for your answer. If the statement is *ever* false, fill in the bubble for False. You do not need to show any work, and there is no partial credit. Each question is worth 2 points.

   (a) If an augmented matrix in reduced row echelon form (RREF) has a pivot in every row, then the corresponding system of linear equations must have exactly one solution.

   ○ True

   ● False

   (b) The vector equation below is consistent for each b in $\mathbf{R}^2$:

   $$x_1 \begin{pmatrix} 1 \\ 3 \end{pmatrix} + x_2 \begin{pmatrix} 4 \\ 4 \end{pmatrix} = b.$$

   ● True

   ○ False

   (c) Suppose u, v, and w are vectors in $\mathbf{R}^3$ with the property that Span$\{u, v\}$ is a plane and Span$\{u, w\}$ is a plane. Then Span$\{v, w\}$ must also be a plane.

   ○ True

   ● False

   (d) If A is a 2×3 matrix and the solution set for $Ax = 0$ is a line, then every vector in $\mathbf{R}^2$ is in the span of the columns of A.

   ● True

   ○ False

   (e) Suppose A is a 3×3 matrix and $A \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$.

   Then the matrix equation $Ax = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ must have infinitely many solutions.

   ● True

   ○ False

Solution: Problem 1.

(a) False: the system might be inconsistent or have infinitely many solutions, as demonstrated by the examples below.

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right), \qquad \left(\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 0 & 1 \end{array}\right).$$

(b) True: the matrix $A = \begin{pmatrix} 1 & 4 \\ 3 & 4 \end{pmatrix}$ has a pivot in every row since it row-reduces in one step to $\begin{pmatrix} 1 & 4 \\ 0 & -8 \end{pmatrix}$, therefore $Ax = b$ is consistent for each b in $\mathbf{R}^2$, which is to say that the equation in question is consistent for each b in $\mathbf{R}^2$.

(c) False: for example, if $u = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $v = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, and $w = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$, then both $\text{Span}\{u, v\}$ and $\text{Span}\{u, w\}$ are the xy-plane in $\mathbf{R}^3$, but $\text{Span}\{v, w\}$ is a line.

(d) True: A is a 2×3 matrix and the solution set to $Ax = 0$ has exactly one free variable (since the sol. set is a line). This means there is exactly one column in A without a pivot, therefore A has 2 pivots, meaning A has a pivot in every row. Therefore, $Ax = b$ is consistent for all b in $\mathbf{R}^2$, which means every b in $\mathbf{R}^2$ is in the span of the columns of A.

(e) True: The homogeneous matrix equation $Ax = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ automatically has the trivial solution $x = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$, and we are also given that $A \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$. Therefore, the homogeneous equation has more than 1 solution, so it must have infinitely many solutions.

2. **Full solutions are on the next page.**

(a) (3 points) Which of the following equations are linear equations in the variables x, y, and z? Clearly fill in the bubble for all that apply.

● $x + \sin\left(\frac{\pi}{7}\right) y + 10z = -2$

○ $x - yz = 1$

● $x - y + z = 3$

(b) (3 points) Which of the following matrices are in reduced row echelon form? Clearly fill in the bubble for all that apply.

○ $\left(\begin{array}{cccc|c} 1 & 0 & 2 & 3 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$

● $\left(\begin{array}{cccc|c} 0 & 0 & 1 & 5 & -1 \end{array}\right)$

● $\left(\begin{array}{cccc|c} 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right)$

(c) (2 pts) Suppose a **consistent** linear system corresponds to an augmented matrix with 7 rows, 5 total columns (including its rightmost column), and 3 pivots in its RREF.

(i) Geometrically, what is the solution set for the system of equations? Clearly fill in the **one** correct bubble for your answer.

○ no solutions ○ a point ● a line ○ a plane

○ all of $\mathbf{R}^2$ ○ all of $\mathbf{R}^3$

(ii) Where does the solution set live?

○ $\mathbf{R}^3$ ● $\mathbf{R}^4$ ○ $\mathbf{R}^5$ ○ $\mathbf{R}^6$ ○ $\mathbf{R}^7$

(d) (2 points) Compute $\begin{pmatrix} 1 & 2 \\ 0 & 2 \\ -1 & 5 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \end{pmatrix}$. Clearly fill in the bubble for your answer.

● $\begin{pmatrix} 3 \\ 4 \\ 11 \end{pmatrix}$ ○ $\begin{pmatrix} -1 & 4 \\ 0 & 4 \\ 1 & 10 \end{pmatrix}$ ○ $\begin{pmatrix} 5 \\ 4 \\ 9 \end{pmatrix}$ ○ $\begin{pmatrix} 3 \\ 0 \\ 11 \end{pmatrix}$ ○ none of these

Solution: Problem 2.

(a) (i) is linear and so is (iii), but (ii) is not linear due to the "yz" term.

(b) (i) is not in RREF because the pivot in the second row has a nonzero term directly above it. However, (ii) and (iii) are in RREF.

(c)
- The augmented matrix has 4 columns **to the left of the vertical bar**, so the corresponding system has 4 variables, which means the solution set lives in $\mathbf{R}^4$.
- The augmented matrix has has 3 pivots that are also all to the left of the vertical bar (since the system is consistent), so there is exactly one free variable in the solution set.

Putting this together, we see the solution set is a line in $\mathbf{R}^4$.

(d)

$$\begin{pmatrix} 1 & 2 \\ 0 & 2 \\ -1 & 5 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \end{pmatrix} = -1 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + 2 \begin{pmatrix} 2 \\ 2 \\ 5 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 4 \\ 4 \\ 10 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 11 \end{pmatrix}.$$

3. On this page, you do not need to show work and there is no partial credit. Parts (a) through (d) are unrelated.

(a) (2 points) Consider the vector equation $x_1 \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + x_2 \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}$.

Which **one** of the following describes its solution set? Clearly fill in the bubble for your answer.

○ no solutions ● a point in $\mathbf{R}^2$ ○ a line in $\mathbf{R}^2$ ○ all of $\mathbf{R}^2$

○ a point in $\mathbf{R}^3$ ○ a line in $\mathbf{R}^3$ ○ a plane in $\mathbf{R}^3$ ○ all of $\mathbf{R}^3$

(b) (2 points) Suppose v_1, v_2, and b are vectors in $\mathbf{R}^3$ with the properties that Span$\{v_1, v_2\}$ is a line and b is **not** in Span$\{v_1, v_2\}$. Which of the following statements are true? Clearly fill in the bubble for all that apply.

● The vector equation $x_1v_1 + x_2v_2 = b$ is inconsistent.

● If w is a vector in $\mathbf{R}^3$ and the vector equation $x_1v_1 + x_2v_2 = w$ is consistent, then it must have infinitely many solutions.

(c) (4 pts) Suppose the solution set to some matrix equation $Ax = b$ has parametric vector form given below, where x_3 is free:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}.$$

(i) Which of these statements must be true? Fill in the bubble for all that apply.

● The matrix equation $Ax = b$ is not homogeneous.

○ The matrix A has 3 rows.

(ii) Which **one** of the following vectors is a solution to $Ax = 0$?

● $\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$ ○ $\begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$

(d) (2 points) Find all values of h (if there are any) so that $\begin{pmatrix} 3 \\ h \end{pmatrix}$ can be written as a linear combination of $\begin{pmatrix} 1 \\ -5 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 10 \end{pmatrix}$.

○ $h = 0$ only ○ $h = 5$ only ○ $h = 10$ only ○ $h = -10$ only

○ $h = 15$ only ● $h = -15$ only ○ all real h ○ none of these

Solution: Problem 3.

(a) The vector equation corresponds to the augmented system

$$\left(\begin{array}{cc|c} 1 & 0 & 1 \\ -2 & 1 & -2 \\ 0 & 1 & 0 \end{array}\right) \xrightarrow{R_2=R_2+2R_1} \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{array}\right) \xrightarrow{R_3=R_3-R_2} \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right).$$

This has the unique solution $x_1 = 1$ and $x_2 = 0$, which is a point in $\mathbf{R}^2$.

(b) (i) By the definitions of span and linear combination, the fact that b is not in Span$\{v_1, v_2\}$ means that b is not a linear combination of v_1 and v_2, which means that the vector equation $x_1v_1 + x_2v_2 = b$ is inconsistent.

(ii) Since Span$\{v_1, v_2\}$ is a line, the matrix A whose columns are v_1 and v_2 has exactly one pivot, which means that any consistent system $\left(\begin{array}{cc|c} v_1 & v_2 & w \end{array}\right)$ has exactly one free variable in its solution set and therefore has infinitely many solutions.

(c) (i) The matrix equation $Ax = b$ is not homogeneous because its solution set does not contain the zero vector, which we can see from the parametric vector form. From the info we are given, we know A must have 3 columns, but it is not necessarily true that A has three rows. For example, if

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 3 \\ 4 \end{pmatrix},$$

then we get the solution set in this problem but A has only two rows.

(ii) From the theory of section 2.4, the free variables are attached to the homogeneous solutions.

- The vector $\begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$ is a particular solution, so $A\begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix} = b$.
- The vector $\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$ is a homogeneous solution, so $A\begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} = 0$.

(d) We row-reduce: $\left(\begin{array}{cc|c} 1 & -2 & 3 \\ -5 & 10 & h \end{array}\right) \xrightarrow{R_2=R_2+5R_1} \left(\begin{array}{cc|c} 1 & -2 & 3 \\ 0 & 0 & h+15 \end{array}\right)$.
This is consistent precisely when $h = -15$.

4. (a) (3 points) Suppose A is an $m \times n$ matrix and b is in $\mathbf{R}^m$. Which of the following conditions **guarantee** that $Ax = b$ is consistent? Clearly fill in the bubble for all that apply.

- ● b is in the span of the columns of A.
- ○ A has a pivot in every column.
- ○ The augmented matrix $\left(A \mid b \right)$ has a pivot in every row.

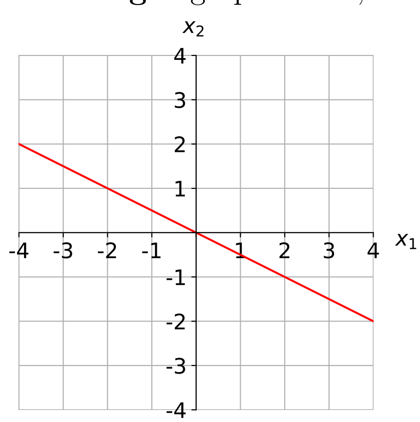
(b) (3 points) Write a set of 3 **different** vectors $\{v_1, v_2, v_3\}$ in $\mathbf{R}^3$ so that Span$\{v_1, v_2, v_3\}$ is a line. Many examples possible, for example

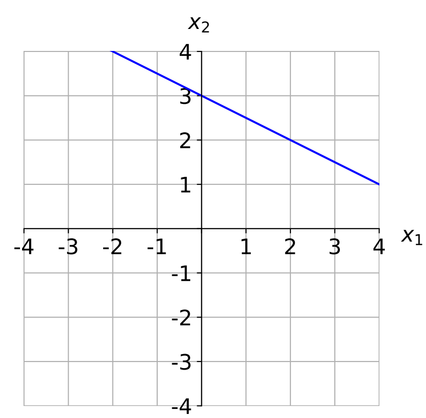
$$v_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad v_2 = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}.$$

(c) (4 points) Suppose b is a nonzero vector in $\mathbf{R}^2$ and A is a 2×2 matrix that satisfies the following conditions:

- The vector $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$ is a solution to $Ax = 0$.
- The vector $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ is a solution to $Ax = b$.

On the **left** graph below, carefully draw the solution set to $Ax = 0$.
On the **right** graph below, carefully draw the solution set to $Ax = b$.





Solution: Problem 4.

(a) (i) is true directly from the definition of span, but (ii) and (iii) are not true. For example, if $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$ and $b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$, then A has a pivot in every column and the augmented matrix $\left(A \,\middle|\, b \right)$ has a pivot in every row, but $Ax = b$ is consistent.

(b) This is essentially verbatim from a past exam and almost identical to one of our worksheet problems. Many examples possible, for example

$$v_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \qquad v_2 = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}, \qquad v_3 = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}.$$

You just need to write three vectors without repeating any of them, so that the matrix $A = \begin{pmatrix} v_1 & v_2 & v_3 \end{pmatrix}$ has one pivot.

(c) First, we note that the 2×2 matrix A has exactly one pivot. For this: we know A has at least one pivot, otherwise, $A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ so $Ax = b$ couldn't be consistent since $b \neq 0$. We also know that A has a non-trivial solution to $Ax = 0$, so A can't have more than one pivot. Therefore, the 2×2 A has exactly one pivot, and the solution set to $Ax = 0$ is a line.

From the theory of section 2.4:

- The solution set to $Ax = 0$ is the line through the origin and $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$, i.e. it is Span$\left\{ \begin{pmatrix} -2 \\ 1 \end{pmatrix} \right\}$.
- The sol. set to $Ax = b$ is the line through $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$ parallel to Span$\left\{ \begin{pmatrix} -2 \\ 1 \end{pmatrix} \right\}$.

For those who are curious: the matrix A and vector b in this example were

$$A = \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} 6 \\ 0 \end{pmatrix}.$$

5. Free response. Show your work! A correct answer without appropriate work will receive little or no credit. For the row-reduction steps you put in part (a) of the problem, you do not need to repeat them in parts (b) and (c).

Consider the system of linear equations in x, y, and z given below:

$$x + y = 0$$

$$y + z = 0$$

$$x + 2cz = 1.$$

(a) Determine all values of c (if there are any) so that the system of equations is inconsistent.

Solution: We row-reduce the corresponding augmented matrix.

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 2c & 1 \end{array}\right) \xrightarrow{R_3 = R_3 - R_1} \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & 2c & 1 \end{array}\right) \xrightarrow{R_3 = R_3 + R_2} \left(\begin{array}{ccc|c} \boxed{1} & 1 & 0 & 0 \\ 0 & \boxed{1} & 1 & 0 \\ 0 & 0 & 2c+1 & 1 \end{array}\right).$$

The entire problem comes down to the final row $\left(\begin{array}{ccc|c} 0 & 0 & 2c+1 & 1 \end{array}\right)$.

This system is inconsistent if and only if it has a pivot in the rightmost column, which means that $2c + 1 = 0$, so $\boxed{c = -\frac{1}{2}}$.

(b) Determine all values of c (if there are any) so that the system of equations has exactly one solution.

Solution: To have exactly one solution, we must have a pivot in every column except the rightmost column, so $2c + 1$ must be a pivot spot in the row $\left(\begin{array}{ccc|c} 0 & 0 & 2c+1 & 1 \end{array}\right)$. Therefore, $2c + 1 \neq 0$, thus $\boxed{c \neq -\frac{1}{2}}$.

(c) Determine all values of c (if there are any) so that the system of equations has infinitely many solutions.

Solution: For the system to have infinitely many solutions, there must be a column left of the augment bar that does NOT a pivot, and the rightmost column cannot have a pivot either. Since the first two columns have pivots, this means the third column must not have a pivot. But in this case, the final row of the augmented matrix would be $\left(\begin{array}{ccc|c} 0 & 0 & 0 & 1 \end{array}\right)$, so the system would be inconsistent.

Therefore, there is no value of c so that the system has infinitely many solutions.

6. Free response. Show your work unless otherwise indicated! A correct answer without sufficient work may receive little or no credit.
Consider the following linear system of equations in the variables $x_1,\ x_2,\ x_3,\ x_4$:

$$x_1 - x_2 - 2x_3 = -2$$

$$2x_1 - 2x_2 - 4x_3 + 2x_4 = -8$$

$$-2x_1 + 3x_2 + 4x_3 = 7.$$

(a) (5 points) Write the augmented matrix corresponding to this system, and put the augmented matrix into RREF.

Solution:

$$\left(\begin{array}{cccc|c} 1 & -1 & -2 & 0 & -2 \\ 2 & -2 & -4 & 2 & -8 \\ -2 & 3 & 4 & 0 & 7 \end{array}\right) \xrightarrow[R_3=R_3+2R_1]{R_2=R_2-2R_1} \left(\begin{array}{cccc|c} 1 & -1 & -2 & 0 & -2 \\ 0 & 0 & 0 & 2 & -4 \\ 0 & 1 & 0 & 0 & 3 \end{array}\right)$$

$$\xrightarrow[\text{then } R_3=R_3/2]{R_2\leftrightarrow R_3} \left(\begin{array}{cccc|c} 1 & -1 & -2 & 0 & -2 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 & -2 \end{array}\right)$$

$$\xrightarrow{R_1=R_1+R_2} \left(\begin{array}{cccc|c} \boxed{1} & 0 & -2 & 0 & 1 \\ 0 & \boxed{1} & 0 & 0 & 3 \\ 0 & 0 & 0 & \boxed{1} & -2 \end{array}\right)$$

(b) (4 points) The system is consistent. Write the set of solutions to the system of equations in parametric **vector** form.

Solution: From above we get $x_1 - 2x_3 = 1$, $x_2 = 3$, x_3 is free, and $x_4 = -2$.

$$x_1 = 1 + 2x_3, \quad x_2 = 3, \quad x_3 = x_3 \text{ (any real number)}, \quad x_4 = -2.$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1+2x_3 \\ 3 \\ x_3 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 0 \\ -2 \end{pmatrix} + \begin{pmatrix} 2x_3 \\ 0 \\ x_3 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 0 \\ -2 \end{pmatrix} + x_3 \begin{pmatrix} 2 \\ 0 \\ 1 \\ 0 \end{pmatrix}.$$

(c) (1 pt) Write ***one*** vector x that solves the system of equations. There is no partial credit on this part, so take time to check by hand that your answer is correct, and if it is not correct then check your work above! If you write more than one vector on this part, or if your answer is unclear, you will not receive any credit.

Solution: From (b), we know that $\begin{pmatrix} 1 \\ 3 \\ 0 \\ -2 \end{pmatrix}$ is a solution, and so is

$$\begin{pmatrix} 1 \\ 3 \\ 0 \\ -2 \end{pmatrix} + x_3 \begin{pmatrix} 2 \\ 0 \\ 1 \\ 0 \end{pmatrix} \qquad \text{for any } x_3.$$

For example, if we choose $x_1 = 1$, we get the solution $\begin{pmatrix} 3 \\ 3 \\ 1 \\ -2 \end{pmatrix}$.

If we choose $x_3 = -1$, we get the solution $\begin{pmatrix} -1 \\ 3 \\ -1 \\ -2 \end{pmatrix}$.

There are infinitely many correct answers that can be obtained by choosing any value you wish for x_3.

One answer that is incorrect is $\begin{pmatrix} 2 \\ 0 \\ 1 \\ 0 \end{pmatrix}$, which satisfies the corresponding homogeneous system of equations.

7. Free response. Show your work! A correct answer without sufficient work will receive little or no credit. Parts (a) and (b) are unrelated.

   (a) (5 points) Find all solutions to the matrix equation $\begin{pmatrix} 1 & -2 & 2 \\ 1 & 1 & 2 \end{pmatrix} x = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$.
   Write your answer in parametric form.

Solution: We put the corresponding augmented matrix in RREF:

$$\left(\begin{array}{ccc|c} 1 & -2 & 2 & -1 \\ 1 & 1 & 2 & 5 \end{array}\right) \xrightarrow{R_2=R_2-R_1} \left(\begin{array}{ccc|c} 1 & -2 & 2 & -1 \\ 0 & 3 & 0 & 6 \end{array}\right) \xrightarrow{R_2=R_2/3} \left(\begin{array}{ccc|c} 1 & -2 & 2 & -1 \\ 0 & 1 & 0 & 2 \end{array}\right)$$
$$\xrightarrow{R_1=R_1+2R_2} \left(\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & 0 & 2 \end{array}\right).$$

This gives us $x_1 + 2x_3 = 3$ and $x_2 = 2$, so the solution set has parametric form

$$x_1 = 3 - 2x_3, \qquad x_2 = 2, \qquad x_3 = x_3 \text{ (any real number)}.$$

   (b) (5 points) Write an augmented matrix in RREF that corresponds to a system of linear equations in the variables x_1, x_2, and x_3 whose solution set has parametric form
$$x_1 = 1 - 3x_3 \qquad x_2 = 2x_3 \qquad x_3 = x_3 \quad (x_3 \text{ any real number}).$$
   Briefly justify why your matrix satisfies these conditions.

Solution: We can rewrite the relationships between x_1, x_2, and x_3 to make them similar to an augmented matrix.

$$\begin{aligned} x_1 = 1 - 3x_3 \quad &\Rightarrow \quad x_1 + 3x_3 = 1 \qquad \left(\begin{array}{ccc|c} 1 & 0 & 3 & 1 \end{array}\right) \\ x_2 = 2x_3 \quad &\Rightarrow \quad x_2 - 2x_3 = 0 \qquad \left(\begin{array}{ccc|c} 0 & 1 & -2 & 0 \end{array}\right) \end{aligned}$$

Putting these together into a single augmented matrix gives us

$$\left(\begin{array}{ccc|c} 1 & 0 & 3 & 1 \\ 0 & 1 & -2 & 0 \end{array}\right),$$

which is in RREF. We can check our answer and see $x_1 = 1 - 3x_3$, $x_2 = 2x_3$, and x_3 is free.

Other answers are possible, for example

$$\left(\begin{array}{ccc|c} 1 & 0 & 3 & 1 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right).$$

This page is reserved ONLY for work that did not fit elsewhere on the exam.

If you use this page, please clearly indicate (on the problem's page and here) which problems you are doing.