Math 1553 Worksheet §2.1, §2.2

1. Consider the system of linear equations

$$\begin{aligned} x + 2y &= 7 \\ 2x + y &= -2 \\ -x - y &= 4. \end{aligned}$$

Question: What are the solutions (if there are any) to the system?

a) Formulate this question as a question about an augmented matrix.

b) Answer the question using row reduction.

c) Formulate this question as a vector equation.

d) What does this question mean in terms of spans?

e) Answer part (d) using the interactive demo.

2. **a)** Write a set of three different vectors whose span is a line in $\mathbf{R}^3$.

b) Write a set of three different vectors whose span is a plane in $\mathbf{R}^3$.

c) Write a set of three vectors whose span is only a single point in $\mathbf{R}^3$.

d) In each of the above questions, if you form the matrix A whose columns are the three vectors, how many pivots does A have?

3. Jameson Locke has challenged you to find a hidden treasure, located at some point (a, b, c). He has honestly guaranteed you that the treasure can be found by starting at the origin and taking steps using

$$v_1 = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix} \qquad v_2 = \begin{pmatrix} 5 \\ -4 \\ -7 \end{pmatrix} \qquad v_3 = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}.$$

By decoding the message, you have discovered that the first and second coordinates of the treasure's location are (in order) -4 and 3.

a) What is the treasure's full location?

b) Give instructions for how to find the treasure by only using v_1, v_2, and v_3. Can you do the same to get the treasure by just using v_1 and v_2?