Math 1553 Worksheet §§3.5-4.3

1. True or false. Answer true if the statement is *always* true. Otherwise, answer false. If your answer is false, either give an example that shows it is false or (in the case of an incorrect formula) state the correct formula.

   a) If A and B are $n \times n$ matrices and both are invertible, then the inverse of AB is $A^{-1}B^{-1}$.

   b) If A is a real 3×3 matrix, then A cannot satisfy $A^2 = -I$.

   c) Suppose A is an $n \times n$ matrix and every vector in $\mathbf{R}^n$ can be written as a linear combination of the columns of A. Then A must be invertible.

   d) If $\det(A) = 1$ and c is a scalar, then $\det(cA) = c \det(A)$.

2. Let $A = \begin{pmatrix} 7 & 1 & 4 & 1 \\ -1 & 0 & 0 & 6 \\ 9 & 0 & 2 & 3 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

a) Compute $\det(A)$.

b) Compute $\det(A^{-1})$ without doing any more work.

c) Compute $\det((A^T)^5)$ without doing any more work.

d) Find the volume of the parallelepiped formed by the columns of A.

3. Suppose we have
$$\det \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = 5.$$

Compute
$$\det \begin{pmatrix} d-3a & e-3b & f-3c \\ a & b & c \\ 2g & 2h & 2i \end{pmatrix}.$$