Math 1553 Worksheet §3.4-3.6

1. True or false. Answer true if the statement is *always* true. Otherwise, answer false. If your answer is false, either give an example that shows it is false or (in the case of an incorrect formula) state the correct formula.

a) If A is an $n \times n$ matrix and the equation $Ax = b$ has at least one solution for each b in $\mathbf{R}^n$, then the solution is *unique* for each b in $\mathbf{R}^n$.

b) If A is a 3×4 matrix and B is a 4×2 matrix, then the linear transformation Z defined by $Z(x) = ABx$ has domain $\mathbf{R}^3$ and codomain $\mathbf{R}^2$.

c) Suppose A is an $n \times n$ matrix and every vector in $\mathbf{R}^n$ can be written as a linear combination of the columns of A. Then A must be invertible.

2. A is $m \times n$ matrix, B is $n \times m$ matrix. Select all correct answers from the box. It is possible to have more than one correct answer.

a) Suppose x is in $\mathbf{R}^m$. Then ABx *must be* in:

$\boxed{\mathrm{Col}(A), \quad \mathrm{Nul}(A), \quad \mathrm{Col}(B), \quad \mathrm{Nul}(B)}$

b) Suppose x in $\mathbf{R}^n$. Then BAx *must be* in:

$\boxed{\mathrm{Col}(A), \quad \mathrm{Nul}(A), \quad \mathrm{Col}(B), \quad \mathrm{Nul}(B)}$

c) If $m > n$, then columns of AB could be linearly $\boxed{independent, \quad dependent}$

d) If $m > n$, then columns of BA could be linearly $\boxed{independent, \quad dependent}$

e) If $m > n$ and $Ax = 0$ has nontrivial solutions, then columns of BA could be linearly $\boxed{independent, \quad dependent}$

3. Consider the following linear transformations:

$$\begin{aligned} T &: \mathbf{R}^3 \longrightarrow \mathbf{R}^2 && T \text{ projects onto the } xy\text{-plane, forgetting the } z\text{-coordinate} \\ U &: \mathbf{R}^2 \longrightarrow \mathbf{R}^2 && U \text{ rotates clockwise by } 90^\circ \\ V &: \mathbf{R}^2 \longrightarrow \mathbf{R}^2 && V \text{ scales the } x\text{-direction by a factor of 2.} \end{aligned}$$

Let A, B, C be the matrices for T, U, V, respectively.

a) Write A, B, and C.

b) Compute the matrix for $U \circ V \circ T$.

c) Describe U^{-1} and V^{-1}, and compute their matrices.