Math 1553 Worksheet §2.5, 2.6, 2.7, 2.9, 3.1

- **1.** If the statement is always true, circle TRUE. Otherwise, circle FALSE. Justify your answer.
 - **a)** Suppose $A = \begin{pmatrix} v_1 & v_2 & v_3 \end{pmatrix}$ and $A \begin{pmatrix} -3 \\ 2 \\ 7 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$. Must v_1, v_2, v_3 be linearly dependent? If true, write a linear dependence relation for the vectors. **TRUE FALSE**

- **b)** In the following, A is an $m \times n$ matrix.
 - (1) **TRUE FALSE** If *A* has linearly independent columns, then Ax = b must have at least one solution for each *b* in \mathbb{R}^m .

(2) **TRUE FALSE** If *b* is a vector in \mathbb{R}^m and Ax = b has exactly one solution, then $m \ge n$.

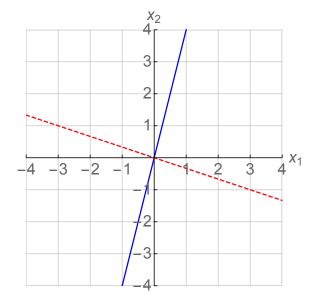
- **2.** Circle **TRUE** if the statement is always true, and circle **FALSE** otherwise.
 - a) If A is a 3×10 matrix with 2 pivots, then dim(NulA) = 8 and rank(A) = 2.

TRUE FALSE

b) If $\{a, b, c\}$ is a basis of a subspace V, then $\{a, a + b, b + c\}$ is a basis of V as well.

TRUE FALSE

3. Write a matrix *A* so that Col(*A*) is the solid blue line and Nul(*A*) is the dotted red line drawn below.



4. Let
$$A = \begin{pmatrix} 1 & -5 & -2 & -4 \\ 2 & 3 & 9 & 5 \\ 1 & 1 & 4 & 2 \end{pmatrix}$$
, and let T be the matrix transformation associated to A , so $T(x) = Ax$.

a) What is the domain of T? What is the codomain of T? Give an example of a vector in the range of *T*.

b) This is extra practice in case the studio finishes the rest of the worksheet early. The RREF of
$$A$$
 is $\begin{pmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$.

- (i) Write bases for Col(A) and Nul(A).
- (ii) Is there a vector in the codomain of T which is not in the range of T? Justify your answer.