Math 1553 Sample Midterm 1, Fall 2024

Name		GT ID	

Circle your instructor and lecture below. Be sure to circle the correct choice!

Jankowski (A, 8:25-9:15) Wessels(B, 8:25-9:15) Hozumi (C, 9:30-10:20)

Wessels (D, 9:30-10:20) Kim (G, 12:30-1:20) Short (H, 12:30-1:20)

Shubin (I, 2:00-2:50) He (L, 3:30-4:20) Wan (M, 3:30-4:20)

Shubin (N, 5:00-5:50) Denton (W, 8:25-9:15)

Please read the following instructions carefully.

- Write your initials at the top of each page.
- The maximum score on this exam is 70 points, and you have 75 minutes to complete this exam. Each problem is worth 10 points.
- There are no calculators or aids of any kind (notes, text, etc.) allowed.
- As always, RREF means "reduced row echelon form."
- The "zero vector" in $\mathbf{R}^n$ is the vector in $\mathbf{R}^n$ whose entries are all zero.
- On free response problems, show your work, unless instructed otherwise. A correct answer without appropriate work may receive little or no credit!
- We will hand out loose scrap paper, but it **will not be graded** under any circumstances. All answers and work must be written on the exam itself, with no exceptions.
- This exam is double-sided. You should have more than enough space to do every problem on the exam, but if you need extra space, you may use the *back side of the very last page of the exam*. If you do this, you must clearly indicate it.
- You may cite any theorem proved in class or in the sections we covered in the text.
- For questions with bubbles to fill in, you must fill them in the correct bubbles clearly and completely or you will not receive credit.

This is a practice exam. It was compiled by combining (and perhaps slightly modifying) two past midterms. It is meant to be similar in format, length, and difficulty to the real exam. It is **not** meant as a comprehensive list of study problems. We recommend completing the practice exam in 75 minutes, without notes or distractions.

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1. TRUE or FALSE. If the statement is *ever* false, circle FALSE. You do not need to show any work, and there is no partial credit. Each question is worth 2 points.

   (a) The set of all solutions (x, y, z) to the following linear equation is a line in $\mathbf{R}^3$:

   $$4x - y + z = 1.$$

   ○ True

   ○ False

   (b) If a consistent system of linear equations has more variables than equations, then it must have infinitely many solutions.

   ○ True

   ○ False

   (c) Suppose v_1, v_2, and b are vectors in $\mathbf{R}^n$ with the property that b is in $\text{Span}\{v_1, v_2\}$. Then the vector $-10b$ must be a linear combination of v_1 and v_2.

   ○ True

   ○ False

   (d) If the RREF of an augmented matrix has final row $\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \end{array}\right)$, then the corresponding system of linear equations must have infinitely many solutions.

   ○ True

   ○ False

   (e) Suppose that A is a 3×3 matrix and there is a vector b in $\mathbf{R}^3$ so that the equation $Ax = b$ has exactly one solution. Then the only solution to the homogeneous equation $Ax = 0$ is the trivial solution.

   ○ True

   ○ False

2. Multiple choice and short answer. You do not need to show work or justify your answers. Parts (a), (b), (c), and (d) are unrelated.

(a) (4 points) Which of the following matrices are in reduced row echelon form? Clearly circle **all** that apply.

○ $\left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 5 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right)$

○ $\left(\begin{array}{ccc|c} 0 & 0 & 1 & -3 \end{array}\right)$

○ $\left(\begin{array}{cccc|c} 1 & -2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right)$

○ $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 10 \\ 0 & 0 & 0 & 0 \end{array}\right)$

(b) (2 points) Consider the vector equation in x and y given by

$$x\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + y\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ -5 \\ 0 \end{pmatrix}.$$

Which **one** of the following describes the solution set to the vector equation?

○ A point in $\mathbf{R}^2$ ○ A line in $\mathbf{R}^2$ ○ All of $\mathbf{R}^2$

○ A point in $\mathbf{R}^3$ ○ A line in $\mathbf{R}^3$ ○ A plane in $\mathbf{R}^3$

(c) (2 points) A system of linear equations in the variables x_1, x_2, x_3 has a solution set with parametric form

$$x_1 = 2 + x_3 \qquad x_2 = x_3 \qquad x_3 = x_3 \quad (x_3 \text{ real}).$$

Which **one** of the following is a solution to the system of equations? Clearly circle your answer.

○ $\begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix}$ ○ $\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ ○ $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ ○ $\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ ○ $\begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix}$

(d) (2 points) Consider a consistent system of three linear equations in four variables. The system corresponds to an augmented matrix whose RREF has two pivots. Complete the following statements by clearly circling the **one** correct answer in each case.

(i) The solution set for the system is:

○ a point ○ a line ○ a plane ○ all of $\mathbf{R}^3$ ○ all of $\mathbf{R}^4$

(ii) Each solution to the system of linear equations is in:

○ $\mathbf{R}$ ○ $\mathbf{R}^2$ ○ $\mathbf{R}^3$ ○ $\mathbf{R}^4$ ○ $\mathbf{R}^5$

3. Short answer. Parts (a) through (d) are unrelated. Briefly show work on part (a).

(a) (2 pts) Find $\begin{pmatrix} 1 & 1 \\ 0 & -3 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix}$. Show work. Enter your answer here: $\begin{pmatrix} \\ \\ \\ \end{pmatrix}$

(b) (2 pts) Let v_1, v_2 and b be nonzero vectors in $\mathbf{R}^n$. Suppose that v_1 and v_2 are **not** scalar multiples of each other and that $c_1v_1 + c_2v_2 = b$ for some real numbers c_1, c_2. Answer the following questions by circling the **one** correct answer.

(i) If $\text{Span}\{v_1, v_2\} = \text{Span}\{v_1, b\}$, then

○ $c_1 = 0$　　○ $c_1 \neq 0$　　○ $c_2 \neq 0$　　○ $c_2 = 0$

(ii) If $\text{Span}\{v_2, b\}$ does **not** contain v_1, then

○ $c_1 = 0$　　○ $c_1 \neq 0$　　○ $c_2 \neq 0$　　○ $c_2 = 0$.

(c) (2 pts) In the spaces below, write 3 **different** vectors v_1, v_2, v_3 in $\mathbf{R}^3$ with the property that $\text{Span}\{v_1, v_2, v_3\}$ is a line.

$v_1 = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$　　$v_2 = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$　　$v_3 = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$

(d) (4 pts) Consider the matrices $A = \begin{pmatrix} 1 & 3 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 100 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{pmatrix}$.

Determine whether the following statements are true or false, and clearly circle the appropriate answer.

(i) The span of the columns of A is $\mathbf{R}^3$.

○ True　○ False

(ii) The span of the columns of B is $\mathbf{R}^2$.

○ True　○ False

(iii) For the vector $b = \begin{pmatrix} 2 \\ 7 \\ 2024 \end{pmatrix}$, the equation $Ax = b$ has exactly one solution.

○ True　○ False

(iv) There is a vector d in $\mathbf{R}^3$ such that the matrix equation $Bx = d$ is inconsistent.

○ True　○ False

4. Short answer. You do not need to show your work on this page. Parts (a), (b), and (c) are unrelated.

(a) (3 points) Find all values of c so that $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ is in Span $\left\{ \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -3 \\ c \end{pmatrix} \right\}$. Clearly circle the **one** correct answer below.

○ $c = 0$ only ○ $c = -3$ only ○ $c = 1$ only ○ $c = 3$ only

○ $c \neq 0$ ○ $c \neq -3$ ○ $c \neq 3$ ○ All real c

(b) (3 pts) Suppose v_1, v_2, v_3, and b are vectors in $\mathbf{R}^n$. Which of the following statements are true? Select all that apply.

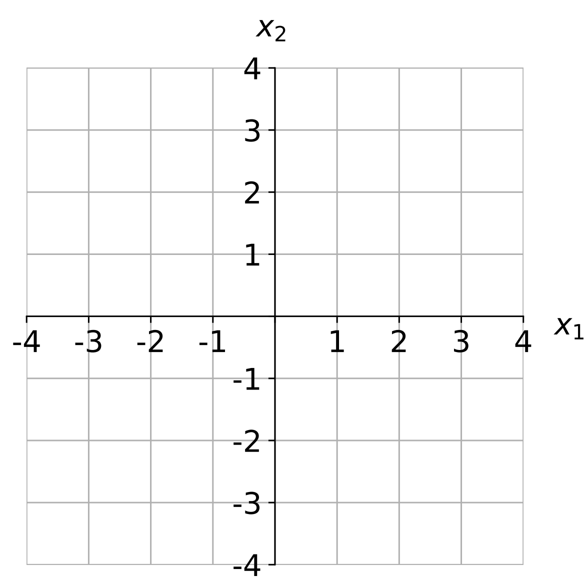
○ If b is in Span$\{v_1, v_2, v_3\}$, then the vector equation $x_1v_1 + x_2v_2 + x_3v_3 = b$ must be consistent.

○ If the vector equation $x_1v_1 + x_2v_2 = b$ is consistent, then the vector equation $x_1v_1 + x_2v_2 + x_3v_3 = b$ must also be consistent.

○ If b is the zero vector, then the vector equation $x_1v_1 + x_2v_2 + x_3v_3 = b$ must be consistent.

(c) (4 points) Suppose A is a 2×2 matrix whose RREF has one pivot, and suppose that $x = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ is one solution to $Ax = 0$.

On the graph below, very carefully draw the set of all solutions to $Ax = 0$.



The rest of the exam is free response. Unless told otherwise, show your work! A correct answer without appropriate work will receive little or no credit, even it is correct.

5. (10 points) Consider the system of linear equations in x and y given by

$$x - 2y = h$$

$$5x + ky = -30,$$

where h and k are real numbers.

(a) Find all values of h and k (if there are any) so that the system is inconsistent.

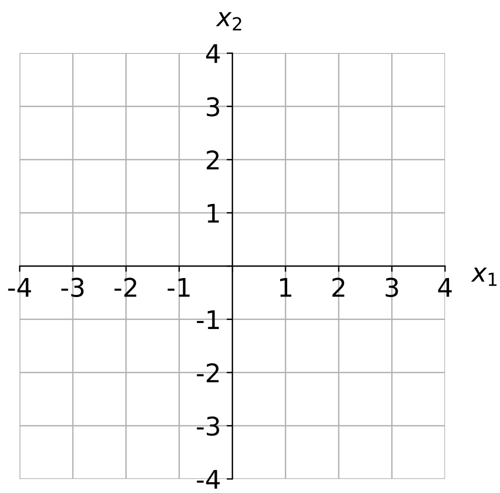
(b) Find all values of h and k (if there are any) so that the system has infinitely many solutions.

(c) Find all values of h and k (if there are any) so that the system has exactly one solution.

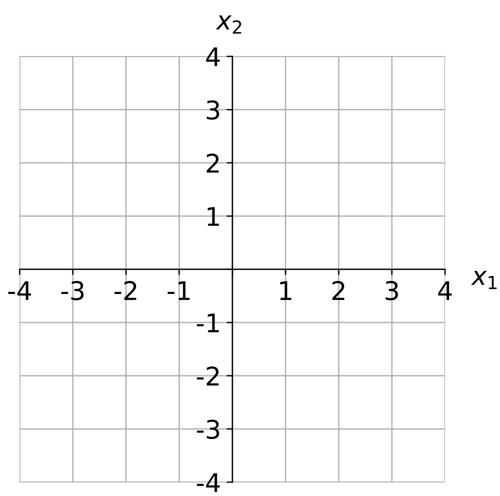
6. Show your work! Parts (a) and (b) are unrelated.

(a) Let $A = \begin{pmatrix} 1 & -2 \\ 4 & -8 \end{pmatrix}$.

(i) (2 points) Draw the span of the columns of A on the graph below.



(ii) (4 points) Draw the solution set for $Ax = 0$ on the graph below.



(b) (4 points) Write an augmented matrix in RREF that corresponds to a system of linear equations in the variables x_1, x_2, and x_3 whose solution set has parametric form

$$x_1 = -1 + 2x_2, \qquad x_2 = x_2 \ \ (x_2 \text{ real}), \qquad x_3 = 0.$$

Free response. Show your work in (a) and (b). You do not need to show work on (c).

7. (10 pts) Consider the following linear system of equations in the variables $x_1,\ x_2,\ x_3,\ x_4$:

$$x_1 + x_2 - 2x_3 + x_4 = 4$$

$$3x_1 + 4x_2 + x_3 + 3x_4 = 5$$

$$-2x_1 - 2x_2 + 4x_3 - x_4 = -3.$$

(a) (5 points) Write the augmented matrix corresponding to this system, and put the augmented matrix into RREF.

(b) (4 points) The system is consistent. Write the set of solutions to the system of equations in parametric vector form.

(c) (1 points) Write **one** vector x that solves the linear system of equations. There is no partial credit on this part, so take time to check by hand that your answer is correct, and if it is not correct then go back and check your work from above!

$$x = \begin{pmatrix} \\ \\ \\ \\ \end{pmatrix}$$

This page is reserved ONLY for work that did not fit elsewhere on the exam.

If you use this page, please clearly indicate (on the problem's page and here) which problems you are doing.