Math 1553 Sample Midterm 1, Fall 2024 SOLUTIONS

Name		**GT ID**	

Circle your instructor and lecture below. Be sure to circle the correct choice!

Jankowski (A, 8:25-9:15) Wessels(B, 8:25-9:15) Hozumi (C, 9:30-10:20)

Wessels (D, 9:30-10:20) Kim (G, 12:30-1:20) Short (H, 12:30-1:20)

Shubin (I, 2:00-2:50) He (L, 3:30-4:20) Wan (M, 3:30-4:20)

Shubin (N, 5:00-5:50) Denton (W, 8:25-9:15)

Please read the following instructions carefully.

- Write your initials at the top of each page.
- The maximum score on this exam is 70 points, and you have 75 minutes to complete this exam. Each problem is worth 10 points.
- There are no calculators or aids of any kind (notes, text, etc.) allowed.
- As always, RREF means "reduced row echelon form."
- The "zero vector" in $\mathbf{R}^n$ is the vector in $\mathbf{R}^n$ whose entries are all zero.
- On free response problems, show your work, unless instructed otherwise. A correct answer without appropriate work may receive little or no credit!
- We will hand out loose scrap paper, but it **will not be graded** under any circumstances. All answers and work must be written on the exam itself, with no exceptions.
- This exam is double-sided. You should have more than enough space to do every problem on the exam, but if you need extra space, you may use the *back side of the very last page of the exam.* If you do this, you must clearly indicate it.
- You may cite any theorem proved in class or in the sections we covered in the text.
- For questions with bubbles to fill in, you must fill them in the correct bubbles clearly and completely or you will not receive credit.

This is a practice exam. It was compiled by combining (and perhaps slightly modifying) two past midterms. It is meant to be similar in format, length, and difficulty to the real exam. It is **not** meant as a comprehensive list of study problems. We recommend completing the practice exam in 75 minutes, without notes or distractions.

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1. TRUE or FALSE. If the statement is *ever* false, circle FALSE. You do not need to show any work, and there is no partial credit. Each question is worth 2 points.

   (a) The set of all solutions (x, y, z) to the following linear equation is a line in $\mathbf{R}^3$:

   $$4x - y + z = 1.$$

   ○ True

   ● False

   (b) If a consistent system of linear equations has more variables than equations, then it must have infinitely many solutions.

   ● True

   ○ False

   (c) Suppose v_1, v_2, and b are vectors in $\mathbf{R}^n$ with the property that b is in Span$\{v_1, v_2\}$. Then the vector $-10b$ must be a linear combination of v_1 and v_2.

   ● True

   ○ False

   (d) If the RREF of an augmented matrix has final row $\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \end{array}\right)$, then the corresponding system of linear equations must have infinitely many solutions.

   ○ True

   ● False

   (e) Suppose that A is a 3×3 matrix and there is a vector b in $\mathbf{R}^3$ so that the equation $Ax = b$ has exactly one solution. Then the only solution to the homogeneous equation $Ax = 0$ is the trivial solution.

   ● True

   ○ False

Solution:

(a) False: the augmented matrix $\left(\begin{array}{ccc|c} 4 & -1 & 1 & 1 \end{array}\right)$ has one pivot, so the solution set has 2 free variables, therefore the equation $4x - y + z = 1$ defines a **plane** in $\mathbf{R}^3$.

(b) True: a **consistent** system of linear equations that has more variables than equations is guaranteed to have at least one free variable and therefore infinitely many solutions.

(c) True: a span of vectors contains all linear combinations of its vectors, so if b is in $\text{Span}\{v_1, v_2\}$ then so is $-10b$. If we wanted to be mathematically precise, we could note that since $c_1v_1 + c_2v_2 = b$ for some c_1 and c_2, it follows that $-10c_1v_1 - 10c_2v_2 = -10b$ so $-10b$ is also in the span of v_1 and v_2.

(d) False: for example, the system for the augmented matrix below no solutions:

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right).$$

(e) True: the solution set to $Ax = b$ is a translation of the solution set to $Ax = 0$ and vice versa, so if $Ax = b$ has just one solution then so does $Ax = 0$, hence the trivial solution $x = 0$ is the only solution to $Ax = 0$.

2. **Full solutions are on the next page.**

(a) (4 points) Which of the following matrices are in reduced row echelon form? Clearly circle **all** that apply.

● $\left(\begin{array}{cccc|c} 1 & 0 & 0 & -1 & 5 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right)$

● $\left(\begin{array}{ccc|c} 0 & 0 & 1 & -3 \end{array}\right)$

○ $\left(\begin{array}{cccc|c} 1 & -2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right)$

● $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 10 \\ 0 & 0 & 0 & 0 \end{array}\right)$

(b) (2 points) Consider the vector equation in x and y given by

$$x \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} + y \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ -5 \\ 0 \end{pmatrix}.$$

Which **one** of the following describes the solution set to the vector equation?

● A point in $\mathbf{R}^2$ ○ A line in $\mathbf{R}^2$ ○ All of $\mathbf{R}^2$

○ A point in $\mathbf{R}^3$ ○ A line in $\mathbf{R}^3$ ○ A plane in $\mathbf{R}^3$

(c) (2 points) A system of linear equations in the variables x_1, x_2, x_3 has a solution set with parametric form

$$x_1 = 2 + x_3 \qquad x_2 = x_3 \qquad x_3 = x_3 \quad (x_3 \text{ real}).$$

Which **one** of the following is a solution to the system of equations? Clearly circle your answer.

○ $\begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix}$ ○ $\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ ○ $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ ○ $\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ ● $\begin{pmatrix} 4 \\ 2 \\ 2 \end{pmatrix}$

(d) (2 points) Consider a consistent system of three linear equations in four variables. The system corresponds to an augmented matrix whose RREF has two pivots. Complete the following statements by clearly circling the **one** correct answer in each case.

(i) The solution set for the system is:

○ a point ○ a line ● a plane ○ all of $\mathbf{R}^3$ ○ all of $\mathbf{R}^4$

(ii) Each solution to the system of linear equations is in:

○ $\mathbf{R}$ ○ $\mathbf{R}^2$ ○ $\mathbf{R}^3$ ● $\mathbf{R}^4$ ○ $\mathbf{R}^5$

Solution:

(a) (i), (ii), and (iv) only.
The matrix in (iii) is not in RREF because the pivot in the second row has a nonzero entry directly above it.

(b) Solving the corresponding augmented system gives us

$$\left(\begin{array}{rr|r} 1 & 0 & 5 \\ -1 & 0 & -5 \\ 0 & 1 & 0 \end{array}\right) \xrightarrow{R_2 = R_2 + R_1} \left(\begin{array}{rr|r} 1 & 0 & 5 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{array}\right) \to \left(\begin{array}{rr|r} 1 & 0 & 5 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

Thus $x = 5$ and $y = 0$, so the only solution is the single point $(5, 0)$ in $\mathbf{R}^2$.

(c) Note (v) is correct because when $x_3 = 2$ we get $x_2 = 2$ and $x_1 = 2 + 2 = 4$.
We see (i) is wrong because when $x_3 = 2$ we need $x_1 = 2 + 2 = 4 \neq 3$, so $\begin{pmatrix} 3 \\ 2 \\ 2 \end{pmatrix}$ is not a solution.
We see (ii) is wrong since when $x_3 = 1$ we need $x_1 = 2 + 1 = 3$.
We see (iii) is wrong since when $x_3 = 0$ we need $x_1 = 2 + 0 = 2$.
We see (iv) is wrong immediately since $x_2 \neq x_3$.

(d) There are 4 variables and the left side of the augment bar matrix has both of the 2 pivots (since the system is consistent), so there are 2 free variables.
(i) Since there are 2 free variables, the solution set is a plane.
(ii) There are 4 variables total, so each solution is in $\mathbf{R}^4$.

3. Full solutions on next page.

(a) (2 pts) Find $\begin{pmatrix} 1 & 1 \\ 0 & -3 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} 4 \\ 2 \end{pmatrix}$. Show work. $\boxed{\begin{pmatrix} 6 \\ -6 \\ 14 \end{pmatrix}}$

(b) (2 pts) Let v_1, v_2 and b be nonzero vectors in $\mathbf{R}^n$. Suppose that v_1 and v_2 are **not** scalar multiples of each other and that $c_1v_1 + c_2v_2 = b$ for some real numbers c_1, c_2. Answer the following questions by circling the **one** correct answer.

(i) If $\text{Span}\{v_1, v_2\} = \text{Span}\{v_1, b\}$, then
○ $c_1 = 0$ ○ $c_1 \neq 0$ ● $c_2 \neq 0$ ○ $c_2 = 0$

(ii) If $\text{Span}\{v_2, b\}$ does **not** contain v_1, then
● $c_1 = 0$ ○ $c_1 \neq 0$ ○ $c_2 \neq 0$ ○ $c_2 = 0$.

(c) (2 pts) In the spaces below, write 3 **different** vectors v_1, v_2, v_3 in $\mathbf{R}^3$ with the property that $\text{Span}\{v_1, v_2, v_3\}$ is a line.
Many possibilities, for example

$$v_1 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \qquad v_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \qquad v_3 = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}.$$

(d) (4 pts) Consider the matrices $A = \begin{pmatrix} 1 & 3 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 100 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{pmatrix}$.
Determine whether the following statements are true or false, and clearly circle the appropriate answer.

(i) The span of the columns of A is $\mathbf{R}^3$.
● True ○ False

(ii) The span of the columns of B is $\mathbf{R}^2$.
○ True ● False

(iii) For the vector $b = \begin{pmatrix} 2 \\ 7 \\ 2024 \end{pmatrix}$, the equation $Ax = b$ has exactly one solution.
● True ○ False

(iv) There is a vector d in $\mathbf{R}^3$ such that the matrix equation $Bx = d$ is inconsistent.
● True ○ False

Solution:

(a) $\begin{pmatrix} 1 & 2 \\ 0 & 5 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 7 \\ 1 \end{pmatrix} = 7 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \\ -7 \end{pmatrix} + \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix} = \begin{pmatrix} 9 \\ 5 \\ -6 \end{pmatrix}.$

(b) Since v_1 and v_2 are nonzero vectors and neither is a scalar multiple of the other, we know $\text{Span}\{v_1, v_2\}$ is a plane.
(i) If $\text{Span}\{v_1, v_2\} = \text{Span}\{v_1, b\}$, then $\text{Span}\{v_1, b\}$ is a plane (from above), so b cannot be a scalar multiple of v_1. This means that $c_2 \neq 0$, because if $c_2 = 0$ then $c_1 v_1 + 0 = b$, which would mean b is a scalar multiple of v_1.
(ii) From the fact $c_1 v_1 + c_2 v_2 = b$ we get $\boxed{c_1 v_1 = b - c_2 v_2}$. Since $\text{Span}\{v_2, b\}$ does not contain v_1, we must have $c_1 = 0$. Otherwise, v_1 would be a linear combination of v_2 and b because the boxed equation would give

$$v_1 = \frac{1}{c_1} b - \frac{c_2}{c_1} v_2.$$

(c) We can just take any three different scalar multiples of the same vector. For example,

$$v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \qquad v_2 = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}, \qquad v_3 = \begin{pmatrix} 5 \\ 5 \\ 5 \end{pmatrix}.$$

(d) (i) True, since A has a pivot in each of its three rows.
(ii) False: the span of the columns of B is a plane in $\mathbf{R}^3$, which is **not** the same as $\mathbf{R}^2$. Every vector in $\mathbf{R}^2$ has two entries, while every vector in the column span of B has three entries. This was nearly copied from slide 12 of the section 1.1 PDF.
(iii) True: $(A|b)$ has a pivot in every column except the rightmost column, so $Ax = b$ has exactly one solution.
(iv) True: for example if $d = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ then $(B|d) = \left(\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$.

4. (a) (3 points) Find all values of c so that $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ is in Span$\left\{ \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \begin{pmatrix} -3 \\ c \end{pmatrix} \right\}$. Clearly circle the **one** correct answer below.

○ $c = 0$ only ○ $c = -3$ only ○ $c = 1$ only ○ $c = 3$ only

○ $c \neq 0$ ○ $c \neq -3$ ● $c \neq 3$ ○ All real c

Solution: We solve the system with augmented matrix below:

$$\left(\begin{array}{cc|c} -1 & -3 & 2 \\ 1 & c & 5 \end{array}\right) \xrightarrow{R_2 = R_2 + R_1} \left(\begin{array}{cc|c} -1 & -3 & 2 \\ 0 & c-3 & 7 \end{array}\right)$$

This is consistent if and only if $c - 3 \neq 0$, so $c \neq 3$.

(b) (3 pts) Suppose v_1, v_2, v_3, and b are vectors in $\mathbf{R}^n$. Which of the following statements are true? Select all that apply.

● If b is in Span$\{v_1, v_2, v_3\}$, then the vector equation $x_1v_1 + x_2v_2 + x_3v_3 = b$ must be consistent.

● If the vector equation $x_1v_1 + x_2v_2 = b$ is consistent, then the vector equation $x_1v_1 + x_2v_2 + x_3v_3 = b$ must also be consistent.

● If b is the zero vector, then the vector equation $x_1v_1 + x_2v_2 + x_3v_3 = b$ must be consistent.

Solution: (i), (ii), and (iii) are all true. Part (i) is a direct consequence of the definition of span, part (ii) is a consequence of the definition of span, and (iii) is a standard fact that we have emphasized.

For (i), if b is in Span$\{v_1, v_2, v_3\}$ then b is a linear combination of v_1, v_2, v_3, which is precisely to say that $x_1v_1 + x_2v_2 + x_3v_3 = b$ is consistent.

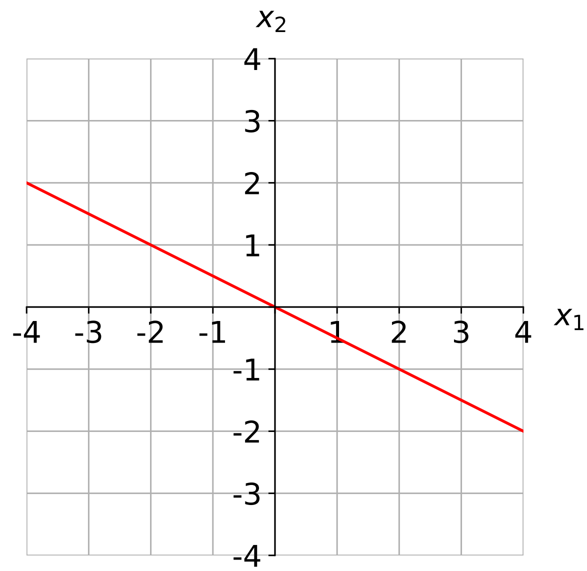
For (ii), note that if $x_1v_1 + x_2v_2 = b$ is consistent then b is already in Span$\{v_1, v_2\}$ which is contained in Span$\{v_1, v_2, v_3\}$, so b is in Span$\{v_1, v_2, v_3\}$. Alternatively, we could see this directly: if $c_1v_1 + c_2v_2 = b$ for some scalars c_1 and c_2 then $c_1v_1 + c_2v_2 + 0v_3 = b$.

For (iii), we have seen it repeatedly emphasized that we can always write the zero vector as a linear combination of any number of vectors:

$$0v_1 + 0v_2 + 0v_3 = 0.$$

(c) (4 points) Suppose A is a 2×2 matrix whose RREF has one pivot, and suppose that $x = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ is one solution to $Ax = 0$.

On the graph below, very carefully draw the set of all solutions to $Ax = 0$.
Solution: A has two columns but one pivot, so $Ax = 0$ has one variable in its solution set. Therefore, the solution set is the line through the origin and $(2, -1)$.



The rest of the exam is free response. Unless told otherwise, show your work! A correct answer without appropriate work will receive little or no credit, even it is correct.

5. (10 points) Consider the system of linear equations in x and y given by

$$x - 2y = h$$

$$5x + ky = -30,$$

where h and k are real numbers.

Before doing any of the three parts, we do one step of row-reduction.

$$\left(\begin{array}{cc|c} 1 & -2 & h \\ 5 & k & -30 \end{array}\right) \xrightarrow{R_2 = R_2 - 5R_1} \left(\begin{array}{cc|c} 1 & -2 & h \\ 0 & k+10 & -30-5h \end{array}\right).$$

(a) Find all values of h and k (if there are any) so that the system is inconsistent.

The system is inconsistent when the rightmost column has a pivot, thus $k+10 = 0$ and $-30 - 5h \neq 0$.

$$\boxed{h \neq -6, \qquad k = -10}.$$

(b) Find all values of h and k (if there are any) so that the system has infinitely many solutions.

The system has infinitely many solutions when the rightmost column does not have a pivot and some other column (in this case, the second) does not either. This means the second row is all zeros, so $k + 10 = 0$ and $-30 - 5h = 0$.

$$\boxed{h = -6, \qquad k = -10}.$$

(c) Find all values of h and k (if there are any) so that the system has exactly one solution.

The system has a unique solution when the system has two pivots and they are both to the left of the augment bar, so $k + 10 \neq 0$ and k can be anything.

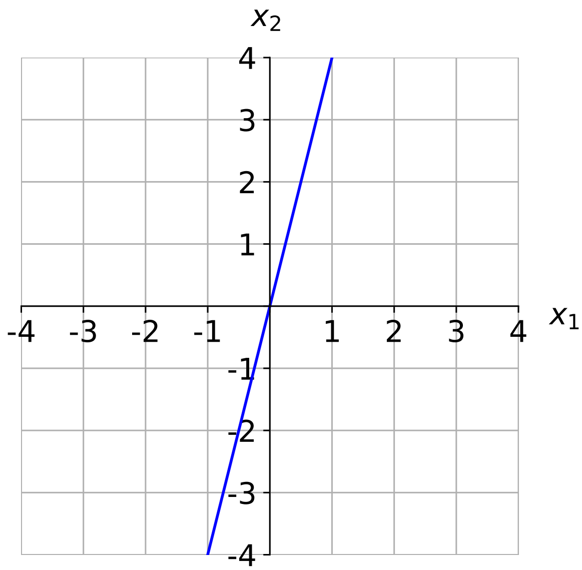
$$\boxed{h \text{ any real number}, \qquad k \neq -10}.$$

6. Show your work! Parts (a) and (b) are unrelated.

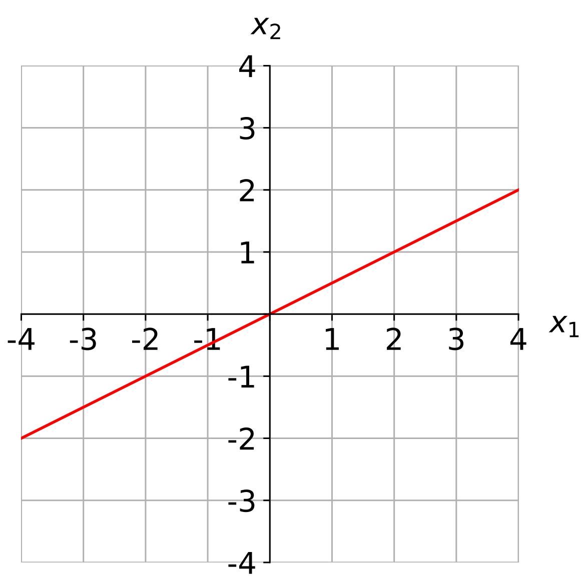
(a) Let $A = \begin{pmatrix} 1 & -2 \\ 4 & -8 \end{pmatrix}$.

(i) (2 points) Draw the span of the columns of A on the graph below.

The span of $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ -8 \end{pmatrix}$ is just the span of $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$, which is the line through the origin containing the point $(1, 4)$.



(ii) (4 points) Draw the solution set for $Ax = 0$ on the graph below.



$\left(A \mid 0 \right)$ row-reduces to $\left(\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right)$ so $x_1 - 2x_2 = 0$, thus $x_1 = 2x_2$ and x_2 is free, so our solution set is the line $x_2 = \dfrac{x_1}{2}$. Alternatively, if we wanted to write this in parametric vector form, we would get $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2x_2 \\ x_2 \end{pmatrix} = x_2 \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, so the solution set is the span of $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

(b) (4 points) Write an augmented matrix in RREF that corresponds to a system of linear equations in the variables x_1, x_2, and x_3 whose solution set has parametric form

$$x_1 = -1 + 2x_2, \qquad x_2 = x_2 \ \ (x_2 \text{ real}), \qquad x_3 = 0.$$

We need three columns to the left of the augment bar because we have three variables, and we need $x_1 - 2x_2 = -1$ with x_2 free and $x_3 = 0$. We can express this by $\left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{array}\right)$ or by equivalent augmented matrices, for example

$$\left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right), \qquad \left(\begin{array}{ccc|c} 1 & -2 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right), \text{etc.}$$

Free response. Show your work in (a) and (b). You do not need to show work on (c).

7. (10 pts) Consider the following linear system of equations in the variables $x_1,\ x_2,\ x_3,\ x_4$:

$$x_1 + x_2 - 2x_3 + x_4 = 4$$

$$3x_1 + 4x_2 + x_3 + 3x_4 = 5$$

$$-2x_1 - 2x_2 + 4x_3 - x_4 = -3.$$

(a) (5 points) Write the augmented matrix corresponding to this system, and put the augmented matrix into RREF.

Solution:

$$\left(\begin{array}{cccc|c} 1 & 1 & -2 & 1 & 4 \\ 3 & 4 & 1 & 3 & 5 \\ -2 & -2 & 4 & -1 & -3 \end{array}\right) \xrightarrow[R_3=R_3+2R_1]{R_2=R_2-3R_1} \left(\begin{array}{cccc|c} 1 & 1 & -2 & 1 & 4 \\ 0 & 1 & 7 & 0 & -7 \\ 0 & 0 & 0 & 1 & 5 \end{array}\right) \xrightarrow{R_1=R_1-R_3} \left(\begin{array}{cccc|c} 1 & 1 & -2 & 0 & -1 \\ 0 & 1 & 7 & 0 & -7 \\ 0 & 0 & 0 & 1 & 5 \end{array}\right)$$

$$\xrightarrow{R_1=R_1-R_2} \left(\begin{array}{cccc|c} 1 & 0 & -9 & 0 & 6 \\ 0 & 1 & 7 & 0 & -7 \\ 0 & 0 & 0 & 1 & 5 \end{array}\right)$$

(b) (4 points) The system is consistent. Write the set of solutions to the system of equations in parametric vector form.

Solution: We get $x_1 - 9x_3 = 6$, $x_2 + 7x_3 = -7$, and $x_4 = 5$ with x_3 free, so the parametric form is

$$x_1 = 6 + 9x_3, \qquad x_2 = -7 - 7x_3, \qquad x_3 = x_3 \quad x_3 \text{ (real)}, \qquad x_4 = 5.$$

This gives

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 6 + 9x_3 \\ -7 - 7x_3 \\ x_3 \\ 5 \end{pmatrix} = \begin{pmatrix} 6 \\ -7 \\ 0 \\ 5 \end{pmatrix} + \begin{pmatrix} 9x_3 \\ -7x_3 \\ x_3 \\ 0 \end{pmatrix} = \begin{pmatrix} 6 \\ -7 \\ 0 \\ 5 \end{pmatrix} + x_3 \begin{pmatrix} 9 \\ -7 \\ 1 \\ 0 \end{pmatrix}.$$

(c) (1 points) Write **one** vector x that solves the linear system of equations. There is no partial credit on this part, so take time to check by hand that your answer is correct, and if it is not correct then go back and check your work from above!

Solution: $x = \begin{pmatrix} 6 \\ -7 \\ 0 \\ 5 \end{pmatrix}$ or $x = \begin{pmatrix} 6+9 \\ -7-7 \\ 0+1 \\ 5+0 \end{pmatrix} = \begin{pmatrix} 15 \\ -14 \\ 1 \\ 5 \end{pmatrix}$ (when $x_3 = 1$) are possible answers, and in fact we can take any value for x_3 and compute x using the answer from part (b). Note that to get credit on this part you **must** write a **correct answer**, you cannot just write an answer that follows your work from (b). This is because the instructions were very specific to check your answer to make sure it solved the system of equations, and that if you found your answer was incorrect then you needed to go back and check your work in (a) and (b) to see where you went wrong.

This page is reserved ONLY for work that did not fit elsewhere on the exam.

If you use this page, please clearly indicate (on the problem's page and here) which problems you are doing.