Supplemental problems: §5.1

1. True or false. Answer true if the statement is always true. Otherwise, answer false.

a) If A and B are $n \times n$ matrices and A is row equivalent to B, then A and B have the same eigenvalues.

b) If A is an $n \times n$ matrix and its eigenvectors form a basis for $\mathbf{R}^n$, then A is invertible.

c) If 0 is an eigenvalue of the $n \times n$ matrix A, then $\text{rank}(A) < n$.

d) The diagonal entries of an $n \times n$ matrix A are its eigenvalues.

e) If A is invertible and 2 is an eigenvalue of A, then $\frac{1}{2}$ is an eigenvalue of A^{-1}.

f) If $\det(A) = 0$, then 0 is an eigenvalue of A.

2. In this problem, you need not explain your answers; just circle the correct one(s).

Let A be an $n \times n$ matrix.

a) Which **one** of the following statements is correct?

1. An eigenvector of A is a vector v such that $Av = \lambda v$ for a nonzero scalar λ.
2. An eigenvector of A is a nonzero vector v such that $Av = \lambda v$ for a scalar λ.
3. An eigenvector of A is a nonzero scalar λ such that $Av = \lambda v$ for some vector v.
4. An eigenvector of A is a nonzero vector v such that $Av = \lambda v$ for a nonzero scalar λ.

b) Which **one** of the following statements is **not** correct?

1. An eigenvalue of A is a scalar λ such that $A - \lambda I$ is not invertible.
2. An eigenvalue of A is a scalar λ such that $(A - \lambda I)v = 0$ has a solution.
3. An eigenvalue of A is a scalar λ such that $Av = \lambda v$ for a nonzero vector v.
4. An eigenvalue of A is a scalar λ such that $\det(A - \lambda I) = 0$.

3. Find a basis $\mathcal{B}$ for the (-1)-eigenspace of $Z = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 2 & 4 \\ 0 & 0 & -1 \end{pmatrix}$

4. Suppose A is an $n \times n$ matrix satisfying $A^2 = 0$. Find all eigenvalues of A. Justify your answer.

5. Match the statements (i)-(v) with the corresponding statements (a)-(e). All matrices are 3×3. There is a unique correspondence. Justify the correspondences in words.

(i) $Ax = \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix}$ has a unique solution.
(ii) The transformation $T(v) = Av$ fixes a nonzero vector.
(iii) A is obtained from B by subtracting the third row of B from the first row of B.
(iv) The columns of A and B are the same; except that the first, second and third columns of A are respectively the first, third, and second columns of B.
(v) The columns of A, when added, give the zero vector.

(a) 0 is an eigenvalue of A.
(b) A is invertible.
(c) $\det(A) = \det(B)$
(d) $\det(A) = -\det(B)$
(e) 1 is an eigenvalue of A.

Supplemental problems: §5.2

1. True or false. If the statement is always true, answer true and justify why it is true. Otherwise, answer false and give an example that shows it is false.

a) If A and B are $n \times n$ matrices with the same eigenvectors, then A and B have the same characteristic polynomial.

b) If A is a 3×3 matrix with characteristic polynomial $-\lambda^3 + \lambda^2 + \lambda$, then A is invertible.

2. Find all values of a so that $\lambda = 1$ an eigenvalue of the matrix A below.

$$A = \begin{pmatrix} 3 & -1 & 0 & a \\ a & 2 & 0 & 4 \\ 2 & 0 & 1 & -2 \\ 13 & a & -2 & -7 \end{pmatrix}$$