Supplemental problems: §1.2, §1.3

1. True or false.

a) If the RREF of an augmented matrix has a pivot in every column, then the corresponding system of linear equations must be consistent.

b) If the RREF of an augmented matrix has a pivot in every column except its rightmost column, then the corresponding system of linear equations has exactly one solution.

c) If the RREF of an augmented matrix has final row $\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \end{array}\right)$, then the corresponding system of linear equations has infinitely many solutions.

2. Is the matrix below in reduced row echelon form?

$$\left(\begin{array}{cccr|c} 1 & 1 & 0 & -3 & 1 \\ 0 & 0 & 1 & -1 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

3. Put an augmented matrix into reduced row echelon form to solve the system

$$x_1 - 2x_2 - 9x_3 + x_4 = 3$$
$$4x_2 + 8x_3 - 24x_4 = 4.$$

4. **a)** Row reduce the following matrices to reduced row echelon form.

b) If these are augmented matrices for a linear system (with the last column being after the $=$ sign), then which are inconsistent? Which have a *unique* solution?

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{pmatrix} \qquad \begin{pmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{pmatrix} \qquad \begin{pmatrix} 3 & -4 & 2 & 0 \\ -8 & 12 & -4 & 0 \\ -6 & 8 & -1 & 0 \end{pmatrix}$$

5. We can use linear algebra to find a polynomial that fits given data, in the same way that we found a circle through three specified points in the Webwork.

Is there a degree-three polynomial $P(x)$ whose graph passes through the points $(-2,6)$, $(-1,4)$, $(1,6)$, and $(2,22)$? If so, how many degree-three polynomials have a graph through those four points? We answer this question in steps below.

a) If $P(x) = a_0 + a_1x + a_2x^2 + a_3x^3$ is a degree-three polynomial passing through the four points listed above, then $P(-2) = 6$, $P(-1) = 4$, $P(1) = 6$, and $P(2) = 22$. Write a system of four equations which we would solve to find a_0, a_1, a_2, and a_3.

b) Write the augmented matrix to represent this system and put it into reduced row-echelon form. Is the system consistent? How many solutions does it have?

6. Consider the linear equation in the variables $x_1,\ x_2$, and x_3 given by

$$x_1 - x_2 + x_3 = 5.$$

If we write the general solution to this system in parametric form, we will find that x_2 and x_3 are free and so

$$x_1 = x_2 - x_3 + 5, \qquad x_2 = x_2 \quad (x_2 \text{ real}), \qquad x_3 = x_3 \quad (x_3 \text{ real}).$$

Is the following also a parametrization of the solution set?

$$x_1 = x_1 \quad (x_1 \text{ real}), \qquad x_2 = x_2 \quad (x_2 \text{ real}), \qquad x_3 = -x_1 + x_2 + 5 \quad (x_3 \text{ real}),$$