Math 1553 Worksheet §5.4-5.6

1. True or false. Justify your answer.

a) A 3×3 matrix A can have a non-real complex eigenvalue with multiplicity 2.

b) It is possible for a 2×2 stochastic matrix to have $-i/2$ as an eigenvalue.

2. Let $A = \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}^{-1}$, and let $x = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \end{pmatrix}$. What happens to $A^n x$ as n gets very large?

3. Let $A = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$. Find all eigenvalues of A. For each eigenvalue, find an associated eigenvector.

4. A video game offers participants the chance to play as one of three characters: Archer, Barbarian, or Cleric. The game has 72 million customers.

In 2022:
Archer is played by 22 million customers.
Barbarian is played by 36 million customers.
Cleric is played by 14 million customers.

One year later, in 2023:

- 50% of the people who started with the Archer still play with the Archer, while 30% have switched to Barbarian and 20% have switched to Cleric.
- 60% of the customers who stared with the Barbarian still play with the Barbarian, while 10% have switched to Archer and 30% have switched to Cleric.
- 70% of the customers who stared with the Cleric still play with the Cleric, while 10% have switched to Archer and 20% have switched to Barbarian.

a) Write down the stochastic matrix A which represents the change in each character's popularity from 2022 to 2023, and use it to find the number of people who played with each character in 2023.

b) Suppose the trend continues each year. In the distant future, what will be the most popular character?

You may use the fact that the 1-eigenspace of A is spanned by $\begin{pmatrix} 6 \\ 13 \\ 17 \end{pmatrix}$.