Supplemental problems: §5.2

1. True or false. If the statement is always true, answer true and justify why it is true. Otherwise, answer false and give an example that shows it is false.

a) If A and B are $n \times n$ matrices with the same eigenvectors, then A and B have the same characteristic polynomial.

b) If A is a 3×3 matrix with characteristic polynomial $-\lambda^3 + \lambda^2 + \lambda$, then A is invertible.

2. Find all values of a so that $\lambda = 1$ an eigenvalue of the matrix A below.

$$A = \begin{pmatrix} 3 & -1 & 0 & a \\ a & 2 & 0 & 4 \\ 2 & 0 & 1 & -2 \\ 13 & a & -2 & -7 \end{pmatrix}$$

3. If A is an $n \times n$ matrix and $\det(A) = 2$, then 2 is an eigenvalue of A.

4. Let $A = \begin{pmatrix} -3 & 0 & -4 \\ 0 & 3 & 0 \\ 6 & 0 & 7 \end{pmatrix}$.

a) Find the eigenvalues of A.

b) Find a basis for each eigenspace of A. Mark your answers clearly.

c) Is there a basis of $\mathbf{R}^3$ that consists of eigenvectors of A? Justify your answer.

Supplemental problems: §5.4

1. True or false. Answer true if the statement is always true. Otherwise, answer false.

a) If A is an invertible matrix and A is diagonalizable, then A^{-1} is diagonalizable.

b) A diagonalizable $n \times n$ matrix admits n linearly independent eigenvectors.

c) If A is diagonalizable, then A has n distinct eigenvalues.

2. Give examples of 2×2 matrices with the following properties. Justify your answers.

a) A matrix A which is invertible and diagonalizable.

b) A matrix B which is invertible but not diagonalizable.

c) A matrix C which is not invertible but is diagonalizable.

d) A matrix D which is neither invertible nor diagonalizable.

3. $A = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 2 & 4 \\ 0 & 0 & -1 \end{pmatrix}$.

a) Find the eigenvalues of A, and find a basis for each eigenspace.

b) Is A diagonalizable? If your answer is yes, find a diagonal matrix D and an invertible matrix C so that $A = CDC^{-1}$. If your answer is no, justify why A is not diagonalizable.

4. Let $A = \begin{pmatrix} 8 & 36 & 62 \\ -6 & -34 & -62 \\ 3 & 18 & 33 \end{pmatrix}$.
The characteristic polynomial for A is $\det(A-\lambda I) = -(\lambda-2)^2(\lambda-3)$. Determine whether A is diagonalizable. If it is, find an invertible matrix C and a diagonal matrix D such that $A = CDC^{-1}$.

5. Which of the following 3×3 matrices are necessarily diagonalizable over the real numbers? (Circle all that apply.)

1. A matrix with three distinct real eigenvalues.
2. A matrix with one real eigenvalue.
3. A matrix with a real eigenvalue λ of algebraic multiplicity 2, such that the λ-eigenspace has dimension 2.
4. A matrix with a real eigenvalue λ such that the λ-eigenspace has dimension 2.

6. Suppose a 2×2 matrix A has eigenvalue $\lambda_1 = -2$ with eigenvector $v_1 = \begin{pmatrix} 3/2 \\ 1 \end{pmatrix}$, and eigenvalue $\lambda_2 = -1$ with eigenvector $v_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

a) Find A.

b) Find A^{100}.

7. Suppose that $A = C \begin{pmatrix} 1/2 & 0 \\ 0 & -1 \end{pmatrix} C^{-1}$, where C has columns v_1 and v_2. Given x and y in the picture below, draw the vectors Ax and Ay.