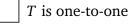
Supplemental problems: §3.2

- **1.** Let *A* be a 3×4 matrix with column vectors v_1, v_2, v_3, v_4 , and suppose $v_2 = 2v_1 3v_4$. Consider the matrix transformation T(x) = Ax.
 - a) Is it possible that *T* is one-to-one? If yes, justify why. If no, find distinct vectors v and w so that T(v) = T(w).
 - **b)** Is it possible that *T* is onto? Justify your answer.
- **2. a)** Which of the following are onto transformations? (Check all that apply.)
 - $T: \mathbf{R}^3 \to \mathbf{R}^3$, reflection over the *xy*-plane
 - $T: \mathbf{R}^3 \to \mathbf{R}^3$, projection onto the *xy*-plane
 - $T: \mathbf{R}^3 \to \mathbf{R}^2$, project onto the *xy*-plane, forget the *z*-coordinate
 - $T: \mathbf{R}^2 \to \mathbf{R}^2$, scale the *x*-direction by 2
 - **b)** Let *A* be a square matrix (square means $n \times n$) and let T(x) = Ax. Which of the following guarantee that *T* is onto? (Check all that apply.)



Ax = 0 is consistent

- 3. Which of the following transformations are **one-to-one**? Circle all that apply.
 - (i) $T : \mathbf{R}^2 \to \mathbf{R}^3$ defined by T(x, y) = (x y, x 2y, x + y).
 - (ii) $T : \mathbf{R}^2 \to \mathbf{R}^2$ which rotates vectors counterclockwise by 17 degrees.

(iii) The transformation T(x) = Ax, where $A = \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{pmatrix}$.

- 4. Which of the following transformations are **onto**? Circle all that apply.
 - (i) $T : \mathbf{R}^3 \to \mathbf{R}^2$ defined by T(x, y, z) = (x + y, x + y).
 - (ii) $T : \mathbf{R}^2 \to \mathbf{R}^2$ which rotates vectors counterclockwise by 17 degrees.

(iii) The transformation T(x) = Ax, where *A* is a 3 × 3 matrix with the property that the equation $Av = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ has exactly one solution.

5. Find all real numbers *h* so that the transformation $T : \mathbf{R}^3 \to \mathbf{R}^2$ given by

$$T(v) = \begin{pmatrix} -1 & 0 & 2-h \\ h & 0 & 3 \end{pmatrix} v$$

is onto.

Supplemental problems: §3.3

- **1.** Circle **T** if the statement is always true, and circle **F** otherwise.
 - a) **T F** If $T : \mathbf{R}^n \to \mathbf{R}^n$ is linear and $T(e_1) = T(e_2)$, then the homogeneous equation T(x) = 0 has infinitely many solutions.
 - b) **T F** If $T : \mathbf{R}^n \to \mathbf{R}^m$ is a one-to-one linear transformation and $m \neq n$, then *T* must not be onto.
- **2.** Consider $T : \mathbf{R}^3 \to \mathbf{R}^3$ given by

$$T(x, y, z) = (x, x + z, 3x - 4y + z, x).$$

Is T one-to-one? Justify your answer.

- **3.** Which of the following transformations *T* are onto? Which are one-to-one? If the transformation is not onto, find a vector not in the range. If the matrix is not one-to-one, find two vectors with the same image.
 - **a)** The transformation $T : \mathbf{R}^3 \to \mathbf{R}^2$ defined by T(x, y, z) = (y, y).
 - **b)** JUST FOR FUN: Consider T: (Smooth functions) \rightarrow (Smooth functions) given by T(f) = f' (the derivative of f). Then T is not a transformation from any \mathbb{R}^n to \mathbb{R}^m , but it is still *linear* in the sense that for all smooth f and g and all scalars c (by properties of differentiation we learned in Calculus 1):

$$T(f+g) = T(f) + T(g)$$
 since $(f+g)' = f' + g'$
 $T(cf) = cT(f)$ since $(cf)' = cf'$.

Is T one-to-one?

- **4.** In each case, determine whether *T* is linear. Briefly justify.
 - **a)** $T(x_1, x_2) = (x_1 x_2, x_1 + x_2, 1).$
 - **b)** $T(x, y) = (y, x^{1/3}).$
 - c) T(x, y, z) = 2x 5z.
- **5.** The second little pig has decided to build his house out of sticks. His house is shaped like a pyramid with a triangular base that has vertices at the points (0,0,0), (2,0,0), (0,2,0), and (1,1,1).

The big bad wolf finds the pig's house and blows it down so that the house is rotated by an angle of 45° in a counterclockwise direction about the *z*-axis (look downward onto the *xy*-plane the way we usually picture the plane as \mathbf{R}^2), and then projected onto the *xy*-plane.

In the worksheet, we found the matrix for the transformation T caused by the wolf. Geometrically describe the image of the house under T.