Math 1553 Worksheet §4.1 - §5.1

1. Let $A = \begin{pmatrix} 7 & 1 & 4 & 1 \\ -1 & 0 & 0 & 6 \\ 9 & 0 & 2 & 3 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

a) Compute $\det(A)$.

b) Compute $\det(A^{-1})$ without doing any more work.

c) Compute $\det((A^T)^5)$ without doing any more work.

d) Find the volume of the parallelepiped formed by the columns of A.

2. Let A be an $n \times n$ matrix.

a) If $\det(A) = 1$ and c is a scalar, what is $\det(cA)$?

b) Using cofactor expansion, explain why $\det(A) = 0$ if A has adjacent identical columns.

3. In what follows, T is a linear transformation with matrix A. Find the eigenvectors and eigenvalues of A without doing any matrix calculations. (Draw a picture!)

a) $T : \mathbf{R}^3 \to \mathbf{R}^3$ that projects vectors onto the xz-plane in $\mathbf{R}^3$.

b) $T : \mathbf{R}^2 \to \mathbf{R}^2$ that reflects vectors over the line $y = 2x$ in $\mathbf{R}^2$.

4. True or false. If the statement is always true, answer true and justify why it is true. Otherwise, answer false and give an example that shows it is false. In every case, assume that A is an $n \times n$ matrix.

a) The entries on the main diagonal of A are the eigenvalues of A.

b) The number λ is an eigenvalue of A if and only if there is a nonzero solution to the equation $(A - \lambda I)x = 0$.

c) To find the eigenvectors of A, we reduce the matrix A to row echelon form.

d) If A is invertible and 2 is an eigenvalue of A, then $\frac{1}{2}$ is an eigenvalue of A^{-1}.

e) If Nul(A) has dimension at least 1, then 0 is an eigenvalue of A and Nul(A) is the 0-eigenspace of A.