Math 1553 Worksheet §§3.5-4.1

1. True or false (justify your answer). Answer true if the statement is *always* true. Otherwise, answer false.

a) If A and B are $n \times n$ matrices and both are invertible, then the inverse of AB is $A^{-1}B^{-1}$.

b) If A is an $n \times n$ matrix and the equation $Ax = b$ has at least one solution for each b in $\mathbf{R}^n$, then the solution is *unique* for each b in $\mathbf{R}^n$.

c) If A is an $n \times n$ matrix and the equation $Ax = b$ has at most one solution for each b in $\mathbf{R}^n$, then the solution must be *unique* for each b in $\mathbf{R}^n$.

d) If A and B are invertible $n \times n$ matrices, then $A+B$ is invertible and $(A+B)^{-1} = A^{-1} + B^{-1}$.

e) If A and B are $n \times n$ matrices and $ABx = 0$ has a unique solution, then $Ax = 0$ has a unique solution.

f) If A is a 3×4 matrix and B is a 4×2 matrix, then the linear transformation Z defined by $Z(x) = ABx$ has domain $\mathbf{R}^3$ and codomain $\mathbf{R}^2$.

g) Suppose A is an $n \times n$ matrix and every vector in $\mathbf{R}^n$ can be written as a linear combination of the columns of A. Then A must be invertible.

2. **a)** Given A is a 3×3 invertible matrix, describe how to find A^{-1} using row reduction.

b) Given A, B are both 3×3 matrix, not necessarily invertible, Describe how to find all possible 3×3 matrix X that satisfies $AX = B$.

c) What is the relation between the previous two parts of the question.

3. Suppose A is an invertible 3×3 matrix with the following equations hold. Find A.

$$A^{-1}e_1 = \begin{pmatrix} 4 \\ 1 \\ 0 \end{pmatrix}, \quad A^{-1}e_2 = \begin{pmatrix} 3 \\ 2 \\ 0 \end{pmatrix}, \quad A^{-1}e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$

4. Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be rotation *clockwise* by $60°$. Let $U : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation satisfying $U(1,0) = (-2,1)$ and $U(0,1) = (1,0)$.

a) Find the standard matrix for the T and U, and compute the determinant of each matrix.

b) Find the standard matrix for the composition $U \circ T$ using matrix multiplication. Compute the determinant.

c) Find the standard matrix for the composition $T \circ U$ using matrix multiplication. Compute the determinant.

d) Is rotating clockwise by $60°$ and then performing U, the same as first performing U and then rotating clockwise by $60°$?

e) What is the relation between the determinants of these matrices?