Math 1553 Worksheet §3.3, 3.4, and intro to 3.5

1. If A is a 3×5 matrix and B is a 3×2 matrix, which of the following are defined?

a) $A - B$

b) AB

c) $A^T B$

d) $B^T A$

e) A^2

2. A is $m \times n$ matrix, B is $n \times m$ matrix. Select proper answers from the box. Multiple answers are possible

a) Take any vector x in $\mathbf{R}^m$, then ABx *must be* in:

$\text{Col}(A)$,	$\text{Nul}(A)$,	$\text{Col}(B)$,	$\text{Nul}(B)$

b) Take any vector x in $\mathbf{R}^n$, then BAx *must be* in:

$\text{Col}(A)$,	$\text{Nul}(A)$,	$\text{Col}(B)$,	$\text{Nul}(B)$

c) If $m > n$, then columns of AB could be linearly

independent,	*dependent*

d) If $m > n$, then columns of BA could be linearly

independent,	*dependent*

e) If $m > n$ and $Ax = 0$ has nontrivial solutions, then columns of BA could be linearly

independent,	*dependent*

3. Consider the following linear transformations:

$T : \mathbf{R}^3 \longrightarrow \mathbf{R}^2$ T projects onto the xy-plane, forgetting the z-coordinate

$U : \mathbf{R}^2 \longrightarrow \mathbf{R}^2$ U rotates clockwise by $90°$

$V : \mathbf{R}^2 \longrightarrow \mathbf{R}^2$ V scales the x-direction by a factor of 2.

Let A, B, C be the matrices for T, U, V, respectively.

a) Compute A, B, and C.

b) Compute the matrix for $V \circ U \circ T$.

c) Compute the matrix for $U \circ V \circ T$.

d) Describe U^{-1} and V^{-1}, and compute their matrices.
If you have not yet seen inverse matrices in lecture, describe geometrically the transformation U^{-1} that would "undo" U in the sense that $(U^{-1} \circ U)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$. Now, do the same for V.

4. On your computer, go to the Interactive Transformation Challenge! Complete the Zoom, Reflect, and Scale challenges. If you complete a challenge in the optimal number of steps, the interactive demo will congratulate you. See if you can complete each of these challenges in the optimal number of steps.