Math 1553 Worksheet §1.2, §1.3

1. **a)** Circle the 'operations' that are legal to use in row reduction, in other words, the operations that will not change the solution set of an arbitrary linear system.

(1) $R_2 = R_3 - 3R_2$

(2) $R_3 = 3R_3$

(3) $R_1 \longleftrightarrow R_2$

(4) $R_1 = R_2 - R_3$

(5) $R_2 = R_2 - R_1^2$

(6) $R_3 = R_3 - \sqrt{R_1}$

b) Define "column operations" on augmented matrices in the same fashion as for the row operations, for example multiplying a column by 2. Do column operations preserve the solution set of a linear system? If yes, explain why; if no, give an example when column operations don't preserve the solution set.

2. **a)** Which of the following matrices are in row echelon form (REF)? Which are in reduced row echelon form (RREF)?

b) For the matrices that are in REF, which entries are the pivots? What are the pivot columns?

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right) \qquad \left(\begin{array}{cccc|c} 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{array}\right) \qquad \left(\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

c) Why is REF useful, i.e. what information does it reveal about the linear system?

d) How many nonzero entries are there in a pivot column of a matrix that is in RREF?

3. Each matrix below is in RREF. In each case, determine whether the corresponding system of linear equations is consistent, and if so, write the parametric form of the general solution and state how many solutions the system has.

(a) $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array}\right)$, (b) $\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$, (c) $\left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 5 \end{array}\right)$, (d) $\left(\begin{array}{ccc|c} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$

4. Find the parametric form for the solution set of the following system of linear equations in x_1, x_2, and x_3 by putting an augmented matrix into reduced row echelon form. State which variables (if any) are free variables. Describe the solution set geometrically.

$$\begin{aligned} x_1 + 3x_2 + x_3 &= 1 \\ -4x_1 - 9x_2 + 2x_3 &= -1 \\ -3x_2 - 6x_3 &= -3. \end{aligned}$$