Math 1553 Worksheet §6.1 - §6.5

1. True/False

(1) If u is in subspace W, and u is also in $W^{\perp}$, then $u = 0$.

(2) If y is in a subspace W, the orthogonal projection of y onto $W^{\perp}$ is 0.

(3) If x is orthogonal to v and w, then x is also orthogonal to $v - w$.

2. **a)** Find the standard matrix B for proj_L, where $L = \text{Span}\left\{\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}\right\}$.

b) What are the eigenvalues of B? Is B is diagonalizable? If so, find an invertible C and diagonal D so that $B = CDC^{-1}$?

c) Describe the column space and null space of the matrix B in terms of L.

3. $y = \begin{pmatrix} 0 \\ 2 \\ 4 \end{pmatrix}, \quad u_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad u_2 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

(1) Determine whether u_1 and u_2
   (a) are linearly independent
   (b) are orthogonal
   (c) span $\mathbf{R}^3$

(2) Is y in $W = \text{Span}\{u_1, u_2\}$?

(3) Compute the vector w that most closely approximates y within W.

(4) Construct a vector, z, that is in $W^{\perp}$.

(5) Make a rough sketch of W, y, w, and z.

4. **a)** Find the least squares solution $\widehat{x}$ to $Ax = e_1$, where $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{pmatrix}$.

b) Find the best fit line $y = Ax + B$ through the points $(0,0)$, $(1,8)$, $(3,8)$, and $(4,20)$.

c) Set up an equation to find the best fit parabola $y = Ax^2 + Bx + C$ through the points $(0,0)$, $(1,8)$, $(3,8)$, and $(4,20)$.