Math 1553 Worksheet: Fundamentals and §1.1

1. In this problem, we will discuss when you can solve a linear system and when you cannot, by using some examples. Moreover, we explore whether this is related to the number of equations and variables.

(1) If a linear system has 3 equations and 2 unknown variables, is it possible to find a solution? If your answer is yes, give an example. If your answer is maybe, give an example of a consistent system and an example of an inconsistent system.

(2) If a linear system has 2 equations and 3 unknown variables, is it possible to find a solution? If your answer is yes, give an example. If your answer is maybe, give an example of a consistent system and an example of an inconsistent system.

(3) If a linear system has 2 equations and 2 unknown variables, must it have a solution? Please explain.

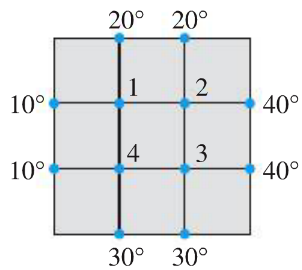
2. Consider the following three planes, where we use (x, y, z) to denote points in $\mathbf{R}^3$:

$$\begin{aligned} 2x + 4y + 4z &= 1 \\ 2x + 5y + 2z &= -1 \\ y + 3z &= 8 \end{aligned}$$

Do all three of the planes intersect? If so, do they intersect at a single point, a line, or a plane?

3. Find all values of h so that the lines $x + hy = -5$ and $2x - 8y = 6$ do *not* intersect. For all such h, draw the lines $x + hy = -5$ and $2x - 8y = 6$ to verify that they do not intersect.

4. The picture below represents the temperatures at four interior nodes of a mesh.



Let $T_1, \ldots, T_4$ be the temperatures at nodes 1 through 4. Suppose that the temperature at each node is the average of the four nearest nodes. For example,

$$T_1 = \frac{10 + 20 + T_2 + T_4}{4}.$$

(1) Write a system of four linear equations whose solution would give the temperatures $T_1, \ldots, T_4$.

(2) Write an augmented matrix that represents that system of equations.