Math 1553 Worksheet §2.6, 2.7, 2.9, 3.1, 3.2

1. Circle **TRUE** if the statement is always true, and circle **FALSE** otherwise.
   a) If $A$ is a $3 \times 10$ matrix with 2 pivots in its RREF, then $\dim(\text{Nul}A) = 8$ and $\text{rank}(A) = 2$.
      
      **TRUE**  **FALSE**

   b) If $A$ is an $m \times n$ matrix and $Ax = 0$ has only the trivial solution, then the transformation $T(x) = Ax$ is onto.
      
      **TRUE**  **FALSE**

   c) If $\{a, b, c\}$ is a basis of a linear space $V$, then $\{a, a + b, b + c\}$ is a basis of $V$ as well.
      
      **TRUE**  **FALSE**

2. Write a matrix $A$ so that $\text{Col}(A)$ is the solid blue line and $\text{Nul}(A)$ is the dotted red line drawn below.

![Diagram of lines](image-url)
3. Let \( A = \begin{pmatrix} 1 & -5 & -2 & -4 \\ 2 & 3 & 9 & 5 \\ 1 & 1 & 4 & 2 \end{pmatrix} \), and let \( T \) be the matrix transformation associated to \( A \), so \( T(x) = Ax \).

a) What is the domain of \( T \)? What is the codomain of \( T \)? Give an example of a vector in the range of \( T \).

b) The RREF of \( A \) is \( \begin{pmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \). Is there a vector in the codomain of \( T \) which is not in the range of \( T \)? Justify your answer.

c) Is \( T \) one-to-one? Is \( T \) onto? Justify your answer.
4. Which of the following transformations $T$ are onto? Which are one-to-one? If the transformation is not onto, find a vector not in the range. If the transformation is not one-to-one, find two vectors with the same image.

a) Counterclockwise rotation by $32^\circ$ in $\mathbb{R}^2$.

b) The transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(x, y, z) = (z, x)$.

c) The transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ defined by $T(x, y, z) = (0, x)$.

d) The matrix transformation with standard matrix $A = \begin{pmatrix} 1 & 6 \\ -1 & 2 \\ 2 & -1 \end{pmatrix}$. 