Problem #1 would be the honor code, so when the problems begin on the next page, the first will be numbered #2.
**Question 2**  
1 pts

If \( \{u, v, w\} \) is a set of linearly dependent vectors, then \( w \) must be a linear combination of \( u \) and \( v \).

- [ ] True
- [ ] False

**Question 3**  
1 pts

Find the value of \( k \) that makes the following vectors linearly dependent:

\[
\begin{pmatrix}
-3 \\
0 \\
3
\end{pmatrix}, \quad
\begin{pmatrix}
3 \\
-3 \\
k
\end{pmatrix}, \quad
\begin{pmatrix}
3 \\
-1
\end{pmatrix}
\]

**Question 4**  
1 pts

If \( \{u, v\} \) is a basis for a subspace \( W \), then \( \{u - v, u + v\} \) is also a basis for \( W \).

- [ ] True
- [ ] False

**Question 5**  
1 pts

Which of the following are subspaces of \( \mathbb{R}^4 \)?

https://gatech.instructure.com/courses/150600/quizzes/138921/take?preview=1
(1) The set $W = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \in \mathbb{R}^4 : 2x - y - z = 0 \right\}$.

(2) The set of solutions to the equation $\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 3 & 0 & -1 \end{pmatrix} x = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

- (2) is a subspace but (1) is not a subspace
- both are subspaces
- neither is a subspace
- (1) is a subspace but (2) is not a subspace

---

**Question 6**

1 pts

Let $W$ be the set of vectors $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ in $\mathbb{R}^3$ with $abc = 0$. Then $W$ is closed under addition, meaning that if $v$ and $w$ are in $W$, then $v + w$ is in $W$.

- True
- False

---

**Question 7**

1 pts

Match the transformations given below with their corresponding $2 \times 2$ matrix.

A. $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$
<table>
<thead>
<tr>
<th>Option</th>
<th>Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.</td>
<td>$\begin{pmatrix} -1 &amp; 0 \ 0 &amp; 1 \end{pmatrix}$</td>
</tr>
<tr>
<td>C.</td>
<td>$\begin{pmatrix} 0 &amp; -1 \ 1 &amp; 0 \end{pmatrix}$</td>
</tr>
<tr>
<td>D.</td>
<td>$\begin{pmatrix} 0 &amp; 1 \ 1 &amp; 0 \end{pmatrix}$</td>
</tr>
<tr>
<td>E.</td>
<td>$\begin{pmatrix} 1 &amp; 0 \ 0 &amp; -1 \end{pmatrix}$</td>
</tr>
</tbody>
</table>

**Counter-clockwise rotation by 90 degrees**: [Choose]

**Reflection about the line y=x**: [Choose]

**Clockwise rotation by 90 degrees**: [Choose]

**Reflection across the x-axis**: [Choose]

**Reflection across the y-axis**: [Choose]

---

**Question 8**

Find the value of $k$ so that the matrix transformation for the following matrix is not onto.

$$\begin{pmatrix} 1 & 3 & 9 \\ 2 & 6 & k \end{pmatrix}$$
### Question 9

Find the **nonzero** value of $k$ that makes the following matrix not invertible.

$$
\begin{pmatrix}
1 & -1 & 0 \\
k & k^2 & 0 \\
-1 & 1 & 5
\end{pmatrix}
$$

*Enter an integer as your answer. Note that 0 is not the correct answer, since the question asks for a nonzero value of $k$."

### Question 10

Match the following definitions with the corresponding term describing a linear transformation $T : \mathbb{R}^m \rightarrow \mathbb{R}^n$.

Each definition should be used exactly once.

A. For each $y$ in $\mathbb{R}^n$ there is at most one $x$ in $\mathbb{R}^m$ so that $T(x) = y$.
B. For each $y$ in $\mathbb{R}^n$ there is at least one $x$ in $\mathbb{R}^m$ so that $T(x) = y$.
C. For each $y$ in $\mathbb{R}^n$ there is exactly one $x$ in $\mathbb{R}^m$ so that $T(x) = y$.
D. For each $x$ in $\mathbb{R}^m$ there is exactly one $y$ in $\mathbb{R}^n$ so that $T(x) = y$.

<table>
<thead>
<tr>
<th>$T$ is a transformation</th>
<th>[ Choose ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$ is one-to-one</td>
<td>[ Choose ]</td>
</tr>
<tr>
<td>$T$ is onto</td>
<td>[ Choose ]</td>
</tr>
<tr>
<td>$T$ is one-to-one and onto</td>
<td></td>
</tr>
</tbody>
</table>
Question 11

Suppose $A$ is a $4 \times 6$ matrix. Then the dimension of the null space of $A$ is at most 2.

- True
- False

Question 12

Complete the entries of the matrix $A$ so that $\text{Col}(A) = \text{Span} \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$ and $\text{Nul}(A) = \text{Span} \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$.

$A = \begin{pmatrix} r & 1 \\ s & 2 \end{pmatrix}$, where $r =$ and $s =$

Question 13

Suppose $T : \mathbb{R}^7 \rightarrow \mathbb{R}^9$ is a linear transformation with standard matrix $A$, and suppose that the range of $T$ has a basis consisting of 3 vectors. What is the dimension of the null space of $A$?
### Question 14

Define a transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^4$ by $T(x, y, z) = (0, x - y, y - x, z)$.

Which one of the following statements is true?

- $T$ is one-to-one but not onto.
- $T$ is neither one-to-one nor onto.
- $T$ is one-to-one and onto.
- $T$ is onto but not one-to-one.

### Question 15

Suppose that $A$ is a $7 \times 5$ matrix, and the null space of $A$ is a line. Say that $T$ is the matrix transformation $T(v) = Av$. Which of the following statements must be true about the range of $T$?

- It is a 6-dimensional subspace of $\mathbb{R}^7$
- It is a 4-dimensional subspace of $\mathbb{R}^7$
- It is a 4-dimensional subspace of $\mathbb{R}^5$
- It is a 6-dimensional subspace of $\mathbb{R}^5$

### Question 16

Say that $S : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ and $T : \mathbb{R}^3 \rightarrow \mathbb{R}^4$ are linear transformations. Which of the following must be true about $T \circ S$?

- It is not onto
Question 17  1 pts

Suppose that \( A \) is an invertible \( n \times n \) matrix. Then \( A + A \) must be invertible.

○ True

○ False

Question 18  1 pts

Suppose \( A \) is a \( 3 \times 3 \) matrix and the equation \( Ax = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix} \) has exactly one solution.

Then \( A \) must be invertible.

○ True

○ False

Question 19  1 pts

Suppose that \( A \) and \( B \) are \( n \times n \) matrices and \( AB \) is not invertible.

Which one of the following statements must be true?
At least one of the matrices A or B is not invertible

- B is not invertible
- None of these
- A is not invertible

### Question 20

Suppose $A$ and $B$ are $3 \times 3$ matrices, with $\det(A) = 3$ and $\det(B) = -6$.

Find $\det(2A^{-1}B)$.

### Question 21

Let $A$ be the $3 \times 3$ matrix satisfying $Ae_1 = e_3$, $Ae_2 = e_2$, and $Ae_3 = 2e_1$ (recall that we use $e_1$, $e_2$, and $e_3$ to denote the standard basis vectors for $\mathbb{R}^3$).

Find $\det(A)$.

### Question 22

Suppose $A$ is a square matrix and $\lambda = -1$ is an eigenvalue of $A$.

Which one of the following statements must be true?

- The columns of $A + I$ are linearly independent.
4/26/2020

Quiz: Final Exam

- The equation $A\mathbf{x} = \mathbf{x}$ has only the trivial solution.
- $A$ is invertible.
- For some nonzero $\mathbf{x}$, the vectors $A\mathbf{x}$ and $\mathbf{x}$ are linearly dependent.
- $\text{Nul}(A + I) = \{0\}$

Question 23

Suppose $A$ is a 4 x 4 matrix with characteristic polynomial $-(1 - \lambda)^2(5 - \lambda)\lambda$.

What is the rank of $A$?

Question 24

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the transformation that reflects across the line $x_2 = 2x_1$.

Find the value of $k$ so that $A \begin{pmatrix} 2 \\ k \end{pmatrix} = \begin{pmatrix} 2 \\ k \end{pmatrix}$.

Question 25

Find the value of $k$ such that the matrix $\begin{pmatrix} 1 & k \\ 1 & 3 \end{pmatrix}$ is not diagonalizable. Enter an integer value below.
Question 26
Suppose that $A$ is a $5 \times 5$ matrix with characteristic polynomial
$(1 - \lambda)^3 (2 - \lambda)(3 - \lambda)$ and also that $A$ is diagonalizable. What is the dimension of
the 1-eigenspace of $A$?

Question 27
Find the value of $t$ such that 3 is an eigenvalue of
\[
\begin{pmatrix}
1 & t & 3 \\
1 & 1 & 1 \\
3 & 0 & 3
\end{pmatrix}
\]
Enter an integer answer below.

Question 28
Say that $A$ is a $2 \times 2$ matrix with characteristic polynomial $(1 - \lambda)(2 - \lambda)$. What is
the characteristic polynomial of $A^2$?

- $(1 - \lambda^2)(4 - \lambda^2)$
- $(1 - \lambda)(4 - \lambda)$
Question 29 1 pts

Suppose that a vector $x$ is an eigenvector of $A$ with eigenvalue 3 and that $x$ is also an eigenvector of $B$ with eigenvalue 4. Which of the following is true about the matrix $2A - B$ and $x$:

- None of these
- $x$ is an eigenvector of $2A - B$ with eigenvalue 2
- $x$ is an eigenvector of $2A - B$ with eigenvalue 3
- $x$ is an eigenvector of $2A - B$ with eigenvalue 1
- $x$ is an eigenvector of $2A - B$ with eigenvalue 4

Question 30 1 pts

Suppose that $A$ is a $4 \times 4$ matrix with eigenvalues 0, 1, and 2, where the eigenvalue 1 has algebraic multiplicity two.

Which of the following must be true?

(1) $A$ is not diagonalizable

(2) $A$ is not invertible

- Both (1) and (2) must be true
- Neither statement is necessarily true
- (1) must be true but (2) might not be true
Question 31

Suppose $A$ is a $5 \times 5$ matrix whose entries are real numbers. Then $A$ must have at least one real eigenvalue.

- True
- False

Question 32

Suppose $A$ is a positive stochastic matrix and $A \begin{pmatrix} 3/5 \\ 2/5 \end{pmatrix} = \begin{pmatrix} 3/5 \\ 2/5 \end{pmatrix}$. Let $v = \begin{pmatrix} 5 \\ 95 \end{pmatrix}$.

As $n$ gets very large, $A^n v$ approaches the vector $\begin{pmatrix} r \\ s \end{pmatrix}$, where:

$r =$  and $s =$  .

Question 33

Suppose that $A$ is a $4 \times 4$ matrix of rank 2. Which one of the following statements must be true?

- none of these
- $A$ is not diagonalizable
Question 34

Suppose $A$ is a $2 \times 2$ matrix whose entries are real numbers, and suppose $A$ has eigenvalue $1 + i$ with corresponding eigenvector \[
\begin{pmatrix}
2 \\
1 + i
\end{pmatrix}.
\]

Which of the following must be true?

- $A$ must have eigenvalue $1 - i$ with corresponding eigenvector \[
\begin{pmatrix}
2 \\
1 + i
\end{pmatrix}.
\]
- $A$ must have eigenvalue $1 + i$ with corresponding eigenvector \[
\begin{pmatrix}
2 \\
1 - i
\end{pmatrix}.
\]
- $A$ must have eigenvalue $1 - i$ with corresponding eigenvector \[
\begin{pmatrix}
2 \\
1 - i
\end{pmatrix}.
\]
- None of these

Question 35

Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear transformation that rotates the plane clockwise by 45 degrees, and let $A$ be the standard matrix for $T$.

Which one of the following statements is true?

- $A$ has one complex eigenvalue with algebraic multiplicity two
- $A$ has one real eigenvalue with algebraic multiplicity two
- $A$ has two distinct real eigenvalues

https://gatech.instructure.com/courses/150600/quizzes/138921/take?preview=1
A has two distinct complex eigenvalues.

Question 36

Suppose \( \mathbf{u} \) and \( \mathbf{v} \) are orthogonal unit vectors (to say that a vector is a unit vector means that it has length 1). Find the dot product

\[
(3\mathbf{u} - 8\mathbf{v}) \cdot 4\mathbf{u}.
\]

Question 37

Find the value of \( k \) that makes the following pair of vectors orthogonal.

\[
\begin{pmatrix} 2 \\ k \\ 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} k \\ 1 \\ -6 \end{pmatrix}
\]

Your answer should be an integer.

Question 38

If \( \mathbf{W} \) is a subspace of \( \mathbb{R}^{100} \) and \( \mathbf{v} \) is a vector in \( \mathbf{W}^\perp \) then the orthogonal projection of \( \mathbf{v} \) to \( \mathbf{W} \) must be the \( \mathbf{0} \) vector.

- True
- False
Question 39  

Suppose $W$ is a subspace of $\mathbb{R}^n$. If $x$ is a vector and $x_W$ is the orthogonal projection of $x$ onto $W$, then $x \cdot x_W$ must be 0.

- True
- False

Question 40  

Suppose that $A$ is a $3 \times 3$ invertible matrix. What is the dot product between the second row of $A$ and third column of $A^{-1}$ equal to?

- 1
- 2
- Not Enough Information is Given
- -2
- -1
- 0

Question 41  

Find the orthogonal projection of $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ onto $\text{Span} \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$. 
The orthogonal projection is \( \begin{pmatrix} a \\ b \end{pmatrix} \), where: \( a = \) and \( b = \).

Enter integers or fractions as your entries.

**Question 42**

Compute the orthogonal projection of the vector \( \begin{pmatrix} 6 \\ 5 \\ 4 \end{pmatrix} \) to the plane spanned by the vectors \( \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \) and \( \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \). What is the first coordinate of the projection? Your answer should be an integer.

**Question 43**

Suppose \( B \) is the standard matrix for the transformation \( T : \mathbb{R}^3 \rightarrow \mathbb{R}^3 \) of orthogonal projection onto the subspace \( W = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in \mathbb{R}^3 \mid x + y + 2z = 0 \right\} \).

What is the dimension of the 1-eigenspace of \( B \)?
Question 44

Let \( W \) be the subspace of \( \mathbb{R}^4 \) given by all vectors \( \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \) such that \( x - y + z + w = 0 \). Find dimension of the orthogonal complement \( W^\perp \).

Question 45

If \( b \) is in the column space of the matrix \( A \) then every solution to \( Ax = b \) is a least squares solution.

○ True
○ False

Question 46

If \( A \) is an \( m \times n \) matrix, \( b \) is in \( \mathbb{R}^m \), and \( \hat{x} \) is a least squares solution to \( Ax = b \), then \( \hat{x} \) is the point in \( \text{Col}(A) \) that is closest to \( b \).

○ True
○ False

Question 47
Find the least squares solution $\hat{x}$ to the linear system

$$
\begin{pmatrix}
6 \\
-2 \\
-2
\end{pmatrix} \mathbf{x} =
\begin{pmatrix}
14 \\
-2 \\
0
\end{pmatrix}.
$$

If your answer is an integer, enter an integer.

If your answer is not an integer, enter a fraction.

---

**Question 48**

Find the best fit line $y = \boxed{} \mathbf{x} + \boxed{}$ for the data points $(-7, -22)$, $(0, -2)$, and $(7, 6)$ using the method of least squares. *Your answers should both be integers.*

---

**Question 49**

Let $\mathbf{A} = 
\begin{pmatrix}
4 & 1 \\
5 & 2
\end{pmatrix} 
\begin{pmatrix}
-3 & 0 \\
0 & -1
\end{pmatrix}
\begin{pmatrix}
4 & 1 \\
5 & 2
\end{pmatrix}^{-1}.

Find $r$ and $s$ so that $\mathbf{A}^3 \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} r \\ s \end{pmatrix}$.

$r = \boxed{}$

$s = \boxed{}$
Question 50

If $A$ is a diagonalizable $6 \times 6$ matrix, then $A$ has 6 distinct eigenvalues.

- True
- False

Question 51

Find the eigenvalues of the matrix $A = \begin{pmatrix} 1 & 4 \\ 4 & 7 \end{pmatrix}$ and write them in increasing order.

The smaller eigenvalue is $\lambda_1 =$ ______.

The larger eigenvalue is $\lambda_2 =$ ______.