Supplemental problems: §5.1

1. True or false. Answer true if the statement is always true. Otherwise, answer false.

a) If A and B are $n \times n$ matrices and A is row equivalent to B, then A and B have the same eigenvalues.

b) If A is an $n \times n$ matrix and its eigenvectors form a basis for $\mathbf{R}^n$, then A is invertible.

c) If 0 is an eigenvalue of the $n \times n$ matrix A, then $\operatorname{rank}(A) < n$.

d) The diagonal entries of an $n \times n$ matrix A are its eigenvalues.

e) If A is invertible and 2 is an eigenvalue of A, then $\dfrac{1}{2}$ is an eigenvalue of A^{-1}.

f) If $\det(A) = 0$, then 0 is an eigenvalue of A.

g) If v and w are eigenvectors of a square matrix A, then so is $v + w$.

2. Find a basis $\mathcal{B}$ for the (-1)-eigenspace of $Z = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 2 & 4 \\ 0 & 0 & -1 \end{pmatrix}$

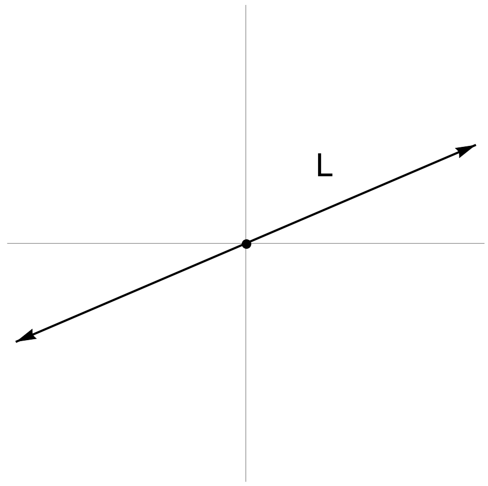
3. Suppose A is an $n \times n$ matrix satisfying $A^2 = 0$. Find all eigenvalues of A. Justify your answer.

4. Match the statements (i)-(v) with the corresponding statements (a)-(e). All matrices are 3×3. There is a unique correspondence. Justify the correspondences in words.

(i) $Ax = \begin{pmatrix} 5 \\ 1 \\ 2 \end{pmatrix}$ has a unique solution.
(ii) The transformation $T(v) = Av$ fixes a nonzero vector.
(iii) A is obtained from B by subtracting the third row of B from the first row of B.
(iv) The columns of A and B are the same; except that the first, second and third columns of A are respectively the first, third, and second columns of B.
(v) The columns of A, when added, give the zero vector.

(a) 0 is an eigenvalue of A.
(b) A is invertible.
(c) $\det(A) = \det(B)$
(d) $\det(A) = -\det(B)$
(e) 1 is an eigenvalue of A.

5. Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the linear transformation which reflects across the line L drawn below, and let A be the standard matrix for T.



a) Write all eigenvalues of A.

b) For each eigenvalue of A, draw one eigenvector on the graph above. Your eigenvector does not need to be perfect, but it should be reasonably accurate.