Supplemental problems: §§2.6, 2.7, 2.9

1. Circle **TRUE** if the statement is always true, and circle **FALSE** otherwise.
   
a) If $A$ is a $3 \times 100$ matrix of rank 2, then $\dim(\text{Nul}A) = 97$.
   
   **TRUE**  **FALSE**

b) If $A$ is an $m \times n$ matrix and $Ax = 0$ has only the trivial solution, then the columns of $A$ form a basis for $\mathbb{R}^m$.
   
   **TRUE**  **FALSE**

c) The set $V = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \in \mathbb{R}^4 \mid x - 4z = 0 \right\}$ is a subspace of $\mathbb{R}^4$.
   
   **TRUE**  **FALSE**

2. Write a matrix $A$ so that $\text{Col}A = \text{Span}\left\{ \begin{pmatrix} 1 \\ -3 \\ 1 \\ 1 \end{pmatrix} \right\}$ and $\text{Nul}A$ is the $xz$-plane.

3. Circle **T** if the statement is always true, and circle **F** otherwise. You do not need to explain your answer.
   
a) If $\{v_1, v_2, v_3, v_4\}$ is a basis for a subspace $V$ of $\mathbb{R}^n$, then $\{v_1, v_2, v_3\}$ is a linearly independent set.
   
   **T**  **F**

b) The solution set of a consistent matrix equation $Ax = b$ is a subspace.
   
   **T**  **F**

c) A translate of a span is a subspace.
   
   **T**  **F**

4. True or false (justify your answer). Answer true if the statement is *always* true. Otherwise, answer false.
   
a) There exists a $3 \times 5$ matrix with rank 4.
   
   **T**

b) If $A$ is an $9 \times 4$ matrix with a pivot in each column, then $\text{Nul}A = \{0\}$.
   
   **T**

c) There exists a $4 \times 7$ matrix $A$ such that nullity $A = 5$.
   
   **T**

d) If $\{v_1, v_2, \ldots, v_n\}$ is a basis for $\mathbb{R}^4$, then $n = 4$.
   
   **T**

5. Find bases for the column space and the null space of

$$A = \begin{pmatrix} 0 & 1 & -3 & 1 & 0 \\ 1 & -1 & 8 & -7 & 1 \\ -1 & -2 & 1 & 4 & -1 \end{pmatrix}.$$
6. Find a basis for the subspace $V$ of $\mathbb{R}^4$ given by

$$V = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \in \mathbb{R}^4 \mid x + 2y - 3z + w = 0 \right\}.$$ 

7. 

a) True or false: If $A$ is an $m \times n$ matrix and $\text{Nul}(A) = \mathbb{R}^n$, then $\text{Col}(A) = \{0\}$.

b) Give an example of a $2 \times 2$ matrix whose column space is the same as its null space.

c) True or false: For some $m$, we can find an $m \times 10$ matrix $A$ whose column span has dimension 4 and whose solution set for $Ax = 0$ has dimension 5.

8. Suppose $V$ is a 3-dimensional subspace of $\mathbb{R}^5$ containing $\begin{pmatrix} 1 \\ -4 \\ 0 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 0 \\ -3 \\ 1 \\ 0 \end{pmatrix}$, and $\begin{pmatrix} 9 \\ 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$.

Is $\left\{ \begin{pmatrix} 1 \\ -4 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 9 \\ 8 \\ 1 \\ 0 \\ 1 \end{pmatrix} \right\}$ a basis for $V$? Justify your answer.

9. 

a) Write a $2 \times 2$ matrix $A$ with rank 2, and draw pictures of $\text{Nul} A$ and $\text{Col} A$.

$$A = \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix} \quad \text{Nul} A = \quad \text{Col} A =$$

b) Write a $2 \times 2$ matrix $B$ with rank 1, and draw pictures of $\text{Nul} B$ and $\text{Col} B$.

$$B = \begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix} \quad \text{Nul} B = \quad \text{Col} B =$$
c) Write a $2 \times 2$ matrix $C$ with rank 0, and draw pictures of $\text{Nul } C$ and $\text{Col } C$.

$$C = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix}$$

$\text{Nul } C =$

$\text{Col } C =$

(In the grids, the dot is the origin.)