Math 1553 Worksheet: Sections 5.1-5.2

1. True or false: If v_1 and v_2 are linearly independent eigenvectors of an $n \times n$ matrix A, then they must correspond to different eigenvalues.

2. In what follows, T is a linear transformation with matrix A. Find the eigenvectors and eigenvalues of A without doing any matrix calculations. (Draw a picture!)

a) $T : \mathbf{R}^3 \to \mathbf{R}^3$ that projects vectors onto the xz-plane in $\mathbf{R}^3$.

b) $T : \mathbf{R}^2 \to \mathbf{R}^2$ that reflects vectors over the line $y = 2x$ in $\mathbf{R}^2$.

3. Answer yes, no, or maybe. Justify your answers. In each case, A is a matrix whose entries are real numbers.

a) Suppose $A = \begin{pmatrix} 3 & 0 & 0 \\ 5 & 1 & 0 \\ -10 & 4 & 7 \end{pmatrix}$. Then the characteristic polynomial of A is

$$\det(A - \lambda I) = (3-\lambda)(1-\lambda)(7-\lambda).$$

b) If A is a 3×3 matrix with characteristic polynomial $-\lambda(\lambda - 5)^2$, then the 5-eigenspace is 2-dimensional.

4. Find the eigenvalues and a basis for each eigenspace of $A = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 2 & 4 \\ 0 & 0 & -1 \end{pmatrix}$.