Math 1553 Worksheet §3.4

1. If A is a 3×5 matrix and B is a 3×2 matrix, which of the following are defined?

a) $A - B$

b) AB

c) $A^T B$

d) A^2

e) $A + I_5$

f) $B^T I_3$

2. Suppose A is an $m \times n$ matrix and B is an $n \times m$ matrix. Select all correct answers from the box. It is possible to have more than one correct answer.

a) Suppose x is in $\mathbf{R}^m$. Then ABx *must be* in:
$\boxed{\text{Col}(A), \quad \text{Nul}(A), \quad \text{Col}(B), \quad \text{Nul}(B)}$

b) If $m > n$, then columns of AB could be linearly $\boxed{\textit{independent}, \quad \textit{dependent}}$

c) If $m > n$, then columns of BA could be linearly $\boxed{\textit{independent}, \quad \textit{dependent}}$

d) If $m > n$ and $Ax = 0$ has nontrivial solutions, then columns of BA could be linearly $\boxed{\textit{independent}, \quad \textit{dependent}}$

3. True or false. Answer true if the statement is *always* true. Otherwise, answer false.

a) If A, B, and C are nonzero 2×2 matrices satisfying $BA = CA$, then $B = C$.

b) Suppose A is an 4×3 matrix whose associated transformation $T(x) = Ax$ is not one-to-one. Then there must be a 3×3 matrix B which is not the zero matrix and satisfies $AB = 0$.

4. Consider the following linear transformations:

$$\begin{array}{ll} T : \mathbf{R}^3 \longrightarrow \mathbf{R}^2 & T \text{ projects onto the } xy\text{-plane, forgetting the } z\text{-coordinate} \\ U : \mathbf{R}^2 \longrightarrow \mathbf{R}^2 & U \text{ rotates clockwise by } 90^\circ \\ V : \mathbf{R}^2 \longrightarrow \mathbf{R}^2 & V \text{ scales the } x\text{-direction by a factor of 2.} \end{array}$$

Let A, B, C be the matrices for T, U, V, respectively.

a) Write A, B, and C.

b) Compute the matrix for $U \circ V \circ T$.

c) Describe U^{-1} and V^{-1}, and compute their matrices.
If you have not yet seen inverse matrices in lecture, describe geometrically the transformation U^{-1} that would “undo" U in the sense that $(U^{-1} \circ U)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$. Now, do the same for V.