Math 1553 Worksheet §5.4, 5.5

1. True or false. If the statement is always true, answer true and justify why it is true. Otherwise, answer false and give an example that shows it is false. If not explicitly stated, assume A is an $n \times n$ matrix.

a) A 3×3 matrix A can have a non-real complex eigenvalue with multiplicity 2.

b) If A is the 3×3 the matrix for the orthogonal projection of vectors in $\mathbf{R}^3$ onto the plane $x + y + z = 0$, then A is diagonalizable.

c) If the RREF of A is diagonalizable, then A must be diagonalizable.

2. Suppose A is a 2×2 matrix satisfying

$$A\begin{pmatrix}-1\\1\end{pmatrix} = \begin{pmatrix}2\\-2\end{pmatrix}, \qquad A\begin{pmatrix}-2\\3\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}.$$

a) Diagonalize A by finding 2×2 matrices C and D (with D diagonal) so that $A = CDC^{-1}$.

b) Find A^{17}.

3. Let $A = \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}^{-1}$, and let $x = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} 3 \\ 1 \end{pmatrix}$. What happens to $A^n x$ as n gets very large?

4. Let $A = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$. Find all eigenvalues of A. For each eigenvalue, find an associated eigenvector.

5. Give an example of matrices A and B which have the same eigenvalues and the same algebraic multiplicities for each eigenvalue, so that A is diagonalizable but B is not diagonalizable. Justify your answer.