

3. Johnny Zenith's video game offers participants the chance to play as one of three characters: Archer, Barbarian, or Cleric. The game has 72 million customers.

In 2019:

Archer is played by 22 million customers.

Barbarian is played by 36 million customers.

Cleric is played by 14 million customers.

One year later, in 2020:

- 50% of the people who started with the Archer still play with the Archer, while 30% have switched to Barbarian and 20% have switched to Cleric.
 - 60% of the customers who started with the Barbarian still play with the Barbarian, while 10% have switched to Archer and 30% have switched to Cleric.
 - 70% of the customers who started with the Cleric still play with the Cleric, while 10% have switched to Archer and 20% have switched to Barbarian.
- a) Write down the stochastic matrix A which represents the change in each character's popularity from 2019 to 2020, and use it to find the number of people who played with each character in 2020.

- b) Suppose the trend continues each year. In the distant future, what will be the most popular character?

You may use the fact that the 1-eigenspace of A is spanned by $\begin{pmatrix} 6 \\ 13 \\ 17 \end{pmatrix}$.