Math 1553 quiz 7 solutions

1.

2. Suppose A is a 4×4 matrix with characteristic polynomial $(3-\lambda)^2(\lambda+1)(\lambda-4)$. Find the determinant of A.

Solution.

Determinant can be computed from product of eigenvalues

$$\det(A) = 3^2 \times (-1) \times 4 = -36$$

3. $A = \begin{pmatrix} 2 & 3 \\ 0 & m \end{pmatrix}$. Find all the values of m so that A is not diagonalizable.

Solution.

$m = 2$ is the only solution. First of all, matrix $\begin{pmatrix} 2 & 3 \\ 0 & 2 \end{pmatrix}$ is not diagonalizable since geometric multiplicity of $\lambda = 2$ is 1. There is not enough eigenvectors.

Suppose $m \neq 2$, then A upper-triangular matrix have two distinct eigenvalues $2, m$ so it must have 2 linearly independent eigenvectors. Then A must be diagonalizable.

4. A is a 4×4 matrix with characteristic polynomial $(1-\lambda)^2(3+\lambda)\lambda$. Is A invertible, diagonalizable? Can you give examples?

Solution.

A has eigenvalues $\lambda = 0, -3, 1, 1$. So $\det(A) = 0$ tells us A is not invertible.

A could be diagonalizable if $\lambda = 1$ have a 2-dimensional eigenspace, for example $A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$. A could be not diagonalizable if $\lambda = 1$ have a 1-dimensional eigenspace, for example $A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$.

5. Suppose $A = \begin{pmatrix} 4 & -3 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 5 & 0 \\ 0 & -3 \end{pmatrix} \begin{pmatrix} 4 & -3 \\ 1 & 1 \end{pmatrix}^{-1}$. Find the value of c so that $A\begin{pmatrix} c \\ 1 \end{pmatrix} = 5\begin{pmatrix} c \\ 1 \end{pmatrix}$

Solution.

$A\begin{pmatrix} c \\ 1 \end{pmatrix} = 5\begin{pmatrix} c \\ 1 \end{pmatrix}$ gives us a eigen-equation $Av = \lambda v$ with $\lambda = 5$ eigenvector $v = \begin{pmatrix} c \\ 1 \end{pmatrix}$

So we know that eigenvalue $\lambda = 5$ have a eigenvector $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$ from the diagonalization, so $c = 4$.

6. Find all real values of a , b , and c so that the matrix A is diagonalizable.

$$A = \begin{pmatrix} -1 & a & b \\ 0 & 3 & c \\ 0 & 0 & 4 \end{pmatrix}$$

Solution.

Since A have 3 distinct eigenvalues, it is always diagonalizable. So a, b, c can take any real number.