Name:______________________________ Studio Section:__________

Math 1553 Quiz 4, Spring 2020 (10 points, 10 minutes)
Jankowski, Lecture C1-C4 (11:15 AM)

Solutions

On this quiz, you do not need to show your work or justify your answers, since every question is either TRUE/FALSE or short answer.

1. True or false. If the statement is always true, circle TRUE. Otherwise, circle FALSE.

a) If W is a 2-dimensional subspace of $\mathbf{R}^5$, then every basis of W must consist of exactly 2 vectors. $\boxed{\text{TRUE}}$ FALSE

b) If A is a 10×15 matrix and the set of solutions to $Ax = 0$ has dimension 8, then Col(A) is a 7-dimensional subspace of $\mathbf{R}^{10}$. $\boxed{\text{TRUE}}$ FALSE

2. (4 points) Consider the matrix A and its RREF, which are given below:

$$A = \begin{pmatrix} 1 & -2 & -1 & 2 \\ 0 & 0 & -2 & 4 \\ 2 & -4 & -4 & 8 \end{pmatrix} \xrightarrow{RREF} \begin{pmatrix} 1 & -2 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

Define a matrix transformation T by $T(x) = Ax$.

a) What is the domain of T? $\mathbf{R}^4$

b) What is the codomain of T? $\mathbf{R}^3$

c) Write a basis for the range of T.

The pivot columns of A form a basis for range(T): $\left\{ \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ -2 \\ -4 \end{pmatrix} \right\}$. In fact, if $A = \begin{pmatrix} | & | & | & | \\ v_1 & v_2 & v_3 & v_4 \\ | & | & | & | \end{pmatrix}$, then any choice of two columns of A will be a basis for range(T) except for $\{v_1, v_2\}$ or $\{v_3, v_4\}$.

turn over for problem 3!

3. (4 points) Match each matrix with its corresponding matrix transformation from $\mathbf{R}^2$ to $\mathbf{R}^2$, which is given by some roman numeral from (i) through (viii).

$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ corresponds to (v) Rotation by 90° counterclockwise.

$B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ corresponds to (i) Reflection across the line $y = x$.

$C = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ corresponds to (vii) Reflection across the x-axis.

$D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ corresponds to (iv) The identity transf. $T(x, y) = (x, y)$.

(i) Reflection across the line $y = x$.
(ii) Reflection across the line $y = -x$.
(iii) Projection onto the x-axis given by $T(x, y) = (x, 0)$
(iv) The identity transformation given by $T(x, y) = (x, y)$.
(v) Rotation by 90° counterclockwise.
(vi) Rotation by 90° clockwise.
(vii) Reflection across the x-axis.
(viii) Reflection across the y-axis.