Name:______________________________ Studio Section:__________

Math 1553 Quiz 3, Spring 2020 (10 points, 10 minutes)
Jankowski, Lecture A1-A3 (8:00 AM)

Solutions

Unless specified otherwise, show your work or you may receive little or no credit, even if your answer is correct.

1. **a)** (2 points) Complete the following definition (be mathematically precise!): Let $v_1, v_2, \dots, v_p$ be vectors in $\mathbf{R}^n$. The *span* of $v_1, v_2, \dots, v_p$, which we denote by Span$\{v_1, \dots, v_p\}$, is...

Solution: In set-builder notation,

$$\text{Span}\{v_1, v_2, \dots, v_p\} = \left\{ x_1 v_1 + \cdots + x_p v_p \;\middle|\; x_1, \dots, x_p \text{ real} \right\}.$$

(the dividing bar could be a colon if desired, and "real" could be "scalar" here; that is just a matter of notation)

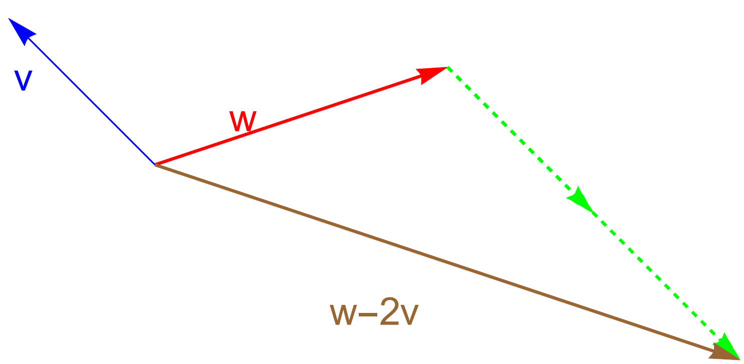
Alternatively, the student could state that Span$\{v_1, v_2, \dots, v_p\}$ is the set of all linear combinations of $v_1, v_2, \dots, v_p$.

b) (1 point) Determine whether the following statement is true or false. Clearly circle your answer (you do not need to show work or justify your answer): If v_1 and v_2 are nonzero vectors in $\mathbf{R}^2$, then Span$\{v_1, v_2\} = \mathbf{R}^2$.

TRUE $\boxed{\text{FALSE}}$ For example, take $v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $v_2 = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$.

2. (3 points) Let v and w be the vectors labeled below. Draw $w - 2v$ as an arrow.

Solution: We use the tip of w and take two steps *backward* in the direction of v.



turn over for problem 3!

3. (4 points) Find all real numbers h (if there are any) so that the following vector equation will have exactly one solution:

$$x_1 \begin{pmatrix} 1 \\ 3 \end{pmatrix} + x_2 \begin{pmatrix} -2 \\ h \end{pmatrix} = \begin{pmatrix} 3 \\ 9 \end{pmatrix}.$$

Solution: We set up the vector equation's corresponding augmented matrix:

$$\left(\begin{array}{cc|c} 1 & -2 & 3 \\ 3 & h & 9 \end{array}\right) \xrightarrow{R_2 = R_2 - 3R_1} \left(\begin{array}{cc|c} 1 & -2 & 3 \\ 0 & h+6 & 0 \end{array}\right).$$

The system must be consistent because the rightmost column of the augmented matrix does not have a pivot.

The system will have exacly one solution precisely when there is a pivot in both columns to the left of the augment, hence $h + 6 \neq 0$, thus $\boxed{h \neq -6}$.