Supplemental problems: §5.5

1. **a)** If A is the matrix that implements rotation by $143°$ in $\mathbf{R}^2$, then A has no real eigenvalues.

b) A 3×3 matrix can have eigenvalues $3, 5$, and $2+i$.

c) If $v = \begin{pmatrix} 2+i \\ 1 \end{pmatrix}$ is an eigenvector of A corresponding to the eigenvalue $\lambda = 1-i$, then $w = \begin{pmatrix} 2i-1 \\ i \end{pmatrix}$ is an eigenvector of A corresponding to the eigenvalue $\lambda = 1-i$.

2. Consider the matrix

$$A = \begin{pmatrix} 3\sqrt{3}-1 & -5\sqrt{3} \\ 2\sqrt{3} & -3\sqrt{3}-1 \end{pmatrix}$$

a) Find both complex eigenvalues of A.

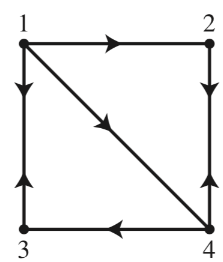
b) Find an eigenvector corresponding to each eigenvalue.

3. This one is just for fun! It demonstrates, by example, that a matrix can have a mix of real and non-real complex eigenvalues, and that we can find a basis for each eigenspace in the usual fashion, even if it takes more work than usual.

Let $A = \begin{pmatrix} 4 & -3 & 3 \\ 3 & 4 & -2 \\ 0 & 0 & 2 \end{pmatrix}$. Find all eigenvalues of A. For each eigenvalue of A, find a corresponding eigenvector.

Supplemental problems: §5.6

1. Suppose the internet has four pages in the following manner. Arrows represent links from one page towards another. For example, page 1 links to page 4 but not vice versa.



a) Write the importance matrix for this internet.

b) Assume there is no damping factor, so the importance matrix is the Google matrix. The 1-eigenspace is spanned by $\begin{pmatrix} 3/4 \\ 3/4 \\ 3/4 \\ 1 \end{pmatrix}$. Find the steady-state vector for the Google matrix. What page has the highest rank?

2. The companies X, Y, and Z fight for customers. This year, company X has 40 customers, Company Y has 15 customers, and Z has 20 customers. Each year, the following changes occur:

- X keeps 75% of its customers, while losing 15% to Y and 10% to Z.
- Y keeps 60% of its customers, while losing 5% to X and 35% to Z.
- Z keeps 65% of its customers, while losing 15% to X and 20% to Y.

Write a stochastic matrix A and a vector x so that Ax will give the number of customers for firms X, Y, and Z (respectively) after one year. You do not need to compute Ax.