**Supplemental problems: §§2.6, 2.7, 2.9, 3.1**

**1.** Circle **T** if the statement is always true, and circle **F** otherwise. You do not need to explain your answer.

**a)** If $\{v_1, v_2, v_3, v_4\}$ is a basis for a subspace $V$ of $\mathbf{R}^n$, then $\{v_1, v_2, v_3\}$ is a linearly independent set.

**b)** The solution set of a consistent matrix equation $Ax = b$ is a subspace.

**c)** A translate of a span is a subspace.

**2.** True or false (justify your answer). Answer true if the statement is *always* true. Otherwise, answer false.

**a)** There exists a $3 \times 5$ matrix with rank 4.

**b)** If $A$ is an $9 \times 4$ matrix with a pivot in each column, then

$$\mathrm{Nul} A = \{0\}.$$

**c)** There exists a $4 \times 7$ matrix $A$ such that nullity $A = 5$.

**d)** If $\{v_1, v_2, \ldots, v_n\}$ is a basis for $\mathbf{R}^4$, then $n = 4$.

**3.** Find bases for the column space and the null space of

$$A = \begin{pmatrix} 0 & 1 & -3 & 1 & 0 \\ 1 & -1 & 8 & -7 & 1 \\ -1 & -2 & 1 & 4 & -1 \end{pmatrix}.$$

**4.** Find a basis for the subspace $V$ of $\mathbf{R}^4$ given by

$$V = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \text{ in } \mathbf{R}^4 \;\middle|\; x + 2y - 3z + w = 0 \right\}.$$

**5.** **a)** True or false: If $A$ is an $m \times n$ matrix and $\mathrm{Nul}(A) = \mathbf{R}^n$, then $\mathrm{Col}(A) = \{0\}$.

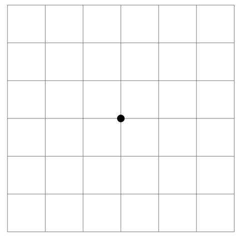
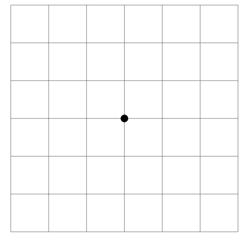
**b)** Give an example of $2 \times 2$ matrix whose column space is the same as its null space.

**c)** True or false: For some $m$, we can find an $m \times 10$ matrix $A$ whose column span has dimension 4 and whose solution set for $Ax = 0$ has dimension 5.

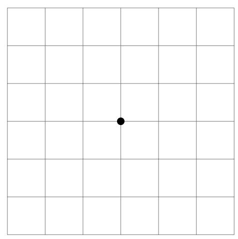
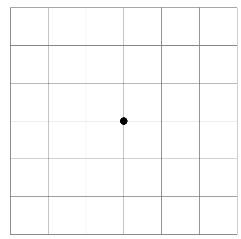
**6.** Suppose $V$ is a 3-dimensional subspace of $\mathbf{R}^5$ containing $\begin{pmatrix} 1 \\ -4 \\ 0 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 0 \\ -3 \\ 1 \\ 0 \end{pmatrix}$, and $\begin{pmatrix} 9 \\ 8 \\ 1 \\ 0 \\ 1 \end{pmatrix}$.

Is $\left\{ \begin{pmatrix} 1 \\ -4 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -3 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 9 \\ 8 \\ 1 \\ 0 \\ 1 \end{pmatrix} \right\}$ a basis for $V$? Justify your answer.

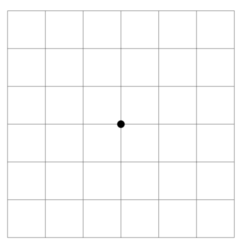
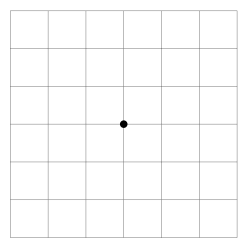
**7.** **a)** Write a $2 \times 2$ matrix $A$ with **rank** 2, and draw pictures of $\mathrm{Nul} A$ and $\mathrm{Col} A$.

$A = \begin{pmatrix} & \\ & \end{pmatrix}$ Nul $A$ =  Col $A$ = 

**b)** Write a $2 \times 2$ matrix $B$ with **rank** 1, and draw pictures of $\mathrm{Nul} B$ and $\mathrm{Col} B$.

$B = \begin{pmatrix} & \\ & \end{pmatrix}$ Nul $B$ =  Col $B$ = 

**c)** Write a $2 \times 2$ matrix $C$ with **rank** 0, and draw pictures of $\mathrm{Nul} C$ and $\mathrm{Col} C$.

$C = \begin{pmatrix} & \\ & \end{pmatrix}$ Nul $C$ =  Col $C$ = 

(In the grids, the dot is the origin.)

**8.** For each matrix $A$, describe what the transformation $T(x) = Ax$ does to $\mathbf{R}^3$ geometrically.

**a)** $\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ **b)** $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. **c)** $\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ **d)** $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$